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PHYSICAL CHARACTERISTICS OF DRAINAGE BASINS

By Bernard L. Golding,¹ M. ASCE and Dana E. Low,² A. M. ASCE

SYNOPSIS

The physical characteristics of major significance, such as; area of the drainage-basin, slope of the principal drainage channel, length of the principal drainage channel, shape of the drainage-basin (basin shape), and general slope of the land in the basin (land slope), are reviewed and defined in terms of recent usage. A standard procedure for plotting the area-distance curves and area-elevation curves is suggested. Also discussed is a new method developed for computation of a modified area-elevation curve. Numerical procedures for computing the various important physical characteristics utilizing the standard area-distance curves and the modified area-elevation curves are included.

INTRODUCTION

Generally, the hydraulic engineer is lucky if there is a stream-gaging station in the immediate vicinity upstream or downstream of the point in which he is interested. Thus in most cases, he is required to estimate stream flow by comparing the drainage-basin above the point of interest with another basin having similar physical characteristics. In the case of a highway, the hydraulic engineer generally computes discharge corresponding to a particular frequency by using a graph for that frequency and entering with an argument in some way

Note.—Discussion open until August 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 3, March, 1960.

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expressive of the physical characteristics of the drainage-basin. These graphs have been developed by correlating discharge against physical characteristics of many similar drainage-basins. In the case of a dam, the unit-hydrograph method is used. Coordinates of the peak value of the unit hydrograph are taken from a graph in which the peak (maximum y-coordinate) and lag (corresponding x-coordinate) of unit hydrographs for basins having similar physical characteristics have been correlated with the physical characteristics of those basins.

Many recent papers have been published using such correlations so that a knowledge of basin characteristics has become an essential for any engineer working in the hydraulics field.

These various physical characteristics of drainage-basins have been defined in many different publications in the past by many different authors.^{3,4,5,6} This has resulted in some of the same named basin characteristics being defined in several different ways. Also, many of these original publications are no longer obtainable as they are out of print. This has made the frequent references to such publications in many recent papers quite useless.

In an attempt to remedy this situation, the various drainage-basin characteristics are reviewed, and a suggested procedure for the computation of the more important characteristics is presented.

AREA OF THE DRAINAGE BASIN

The area of the drainage-basin is its most important physical characteristic. The area is actually the horizontal projection of the land surface from which run off into the surface channels, above the point of interest, occurs. It is ordinarily measured on topographic maps by location of the divide, or separating ridge, which determines whether surface run off will flow toward the basin in question, or flow into adjacent basins. The size of the drainage area is generally expressed in square miles (or acres for small areas). Occasionally, sub-surface drainage-basins (ground-water flow) encompass different areas. However, this complication is very rare and generally not of great significance.

LENGTH OF THE PRINCIPAL DRAINAGE CHANNEL

The length of the principal drainage channel is normally measured along the main channel, from the point of interest to the drainage-basin divide. This length is normally measured as short chords on United States Geological Survey (USGS) quadrangle maps, which along with the projection of the principal channel to the basin divide, may cause some minor discrepancies in the length when measured by different individuals. However, such discrepancies are generally insignificant in their effect on hydrologic computations.

³ "Topographic Characteristics of Drainage Basins," by W. B. Langbein, U. S. Geol. Survey Water Supply Paper 968-C, 1947.

⁴ "Drainage Basin Characteristics," by R. E. Horton, Transactions Amer. Geophysical Union, Vol. 13, 1932, pp. 350-361.

⁵ "Synthetic Unit Graphs," by F. F. Snyder, Transactions Amer. Geophysical Union, Vol. 19, Part 1, 1938, pp. 447-454.

⁶ "Unit-Hydrograph Lag and Peak Flow Related to Basin Characteristics," by A. B. Taylor and H. E. Schwarz, Transactions Amer. Geophysical Union, Vol. 33, 1952, pp. 235-246.

SLOPE OF THE PRINCIPAL DRAINAGE CHANNEL

The slope of the principal drainage channel is probably the most significant physical characteristic after the area of the drainage-basin. The simplest method for expressing the slope of the principal drainage channel is to divide the length of the channel by the difference in elevation between its upper and lower ends (Definition 1, Fig. 1). Generally, the length of the channel is measured beyond the upper end of the clearly discernible stream channel to the drainage divide and then divided by the difference in elevation between this point on the ridge line and the point of interest on the channel. The USGS has used a slope parameter in studies in the New England states in which the length of that part of the stream between points 85% and 10% of the total distance above the point of interest is divided by the difference in elevation at these places.⁷ They are of the opinion that the most upstream part of the slope, in the steep

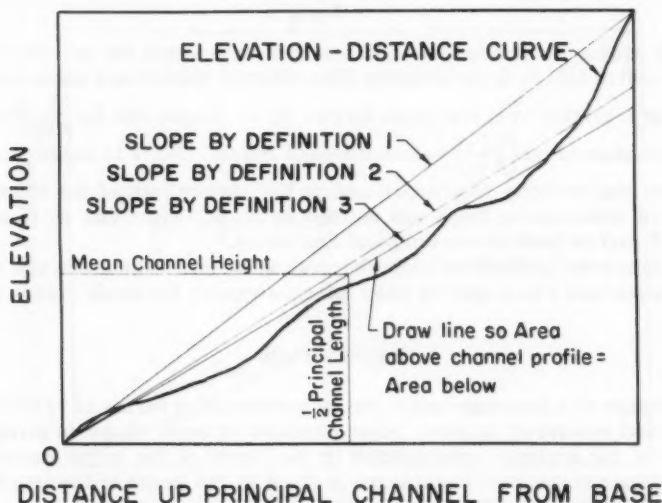


FIG. 1.—SLOPE OF THE PRINCIPAL DRAINAGE CHANNEL

headwaters, might affect the slope out of proportion to the volume of water furnished by the headwater area, and also that the most downstream part of the slope of the principal channel may be so flat that it might not indicate the true portion of the flow which the lower portion of the drainage area actually does contribute.

A slope of the principal drainage channel can also be determined by dividing twice the mean height of the channel profile above the point of interest by the length of the principal channel.

The mean height of the profile is obtained by integration of the elevation-distance curve (channel profile) and dividing by the length of the channel (Definition 3, Fig. 1). The integration can be accomplished by "planimetry" the area below the elevation-distance curve. This slope is also given by the slope

⁷ "Channel-Slope Factor in Flood Frequency Analyses," by M. S. Benson, Proceedings Paper 1994, Journal of the Hydr. Div., ASCE, Vol. 85, No. HY 4, April, 1959, pp. 1-9.

of the line drawn through the origin of the stream-profile curve such that the area under it equals the area under the profile curve. The mean channel height is at a point on the above-cited line at one-half the length of the principal drainage channel measured from either end. Rather than computing mean channel height as such for use in determining channel slope, the mean basin height (to be defined, subsequently, under the heading "Land Slope") is often used and the resultant slope is called the mean basin slope. This effectively introduces a weighting factor to account for slope of the basin as well as slope of the stream channel.

A probable better and more rational definition for slope of the principal drainage channel is the slope of an equivalent stream having the same travel time and same length (Definition 2, Fig. 1). This slope is computed by

$$S_{st} = \frac{p}{\sum p \sqrt{s_i}} \quad \dots \dots \dots (1)$$

where p equals the number of equal reaches into which the stream has been divided (often 10), s_i is the average slope of each such reach measured as the change of elevation over the reach divided by its length, and $\sum p \frac{1}{\sqrt{s_i}}$ represents the summation of the $\left(\frac{1}{\sqrt{s_i}}\right)$ -values for each reach. Eq. 1 is based on the assumption that velocity is proportional to the square root of the slope. This method of determining slope was defined by A. B. Taylor and H. E. Schwarz, M. ASCE and is used by the Corps of Engineers.⁸

The latter two methods of slope determination give values that are approximately equal and which may be used interchangeably for small rivers (Fig. 1).

BASIN SHAPE

The shape of a drainage-basin can be expressed in terms of various easily defined and measured factors. One measure of basin shape is given by the quotient of the drainage area divided by the length of the major watercourse, A/L . Another measure of basin shape is given by the length of the major watercourse squared divided by the basin area L^2/A .

The most commonly used basin shape factors are L_{ca} and the product L times L_{ca} to some power, usually $(L L_{ca})^{0.30}$, where L is the length along the longest watercourse and L_{ca} is often defined as the distance along the main drainage channel from the point of interest to a point opposite the computed center of gravity (centroid) of the drainage area.^{5,6} A more accurate definition of L_{ca} is given subsequently. The term L_{ca} is computed by integration of the area-distance curve and dividing by the drainage area.

To construct the area-distance curve (Fig. 2) the channel length is first subdivided into separate reaches. These reaches may be of equal length along the watercourse, such as 1-mile reaches or equal parts of the total length. Dividing points along the stream could also be made at points where contours cross. A preferable method, especially for larger rivers, is to divide the watercourse length just above and below each major tributary. The areas in this case are the contributory drainage areas at these points. After the channel is divided into reaches, the sub-drainage basins contributing to the stream between the limits of each reach are determined.

⁸ Unit Hydrograph Compilations, Project CW 153, Washington District, Corps of Engrs., Washington, D. C., 3 Volumes 1949, 1 Volume 1954.

A curve of channel distance upstream from the point of interest (or percentage of channel distance) versus area (or percentage of area) contributing within that distance is then drawn as shown in Fig. 2. The distance to the centroid of the area is found by dividing the area above the curve by the drainage area (or percentage of the drainage area) which is the maximum ordinate. As an alternative, L_{ca} may also be found by integrating the area-distance curve and dividing by the drainage area. The integration is accomplished by "planimetry" the area above the area-distance curve.

A suggested method for plotting the area-distance curve and a sample computation of L_{ca} based on this curve will be presented subsequently.

Physical Definition of L_{ca} .—Previously, it was stated that the centroid of the area above the area-distance curve determines L_{ca} . In other words, L_{ca} is the length measured up the stream channel from the base of the drainage area to a point corresponding to the centroid of the area above the area-distance

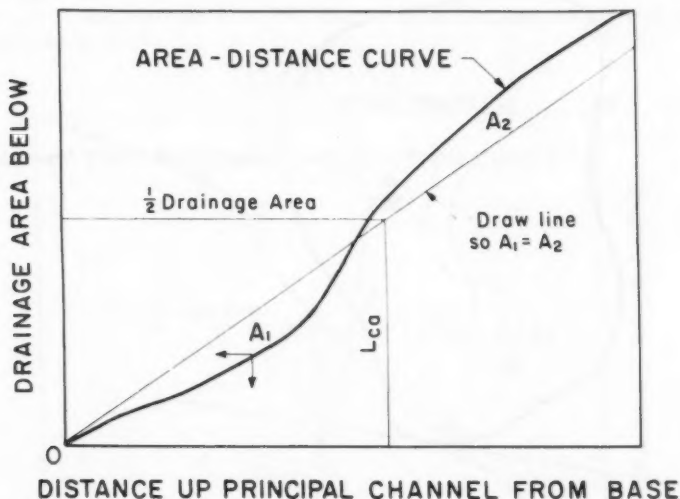


FIG. 2.—BASIN SHAPE FACTOR, L_{ca}

curve. Although all of the various methods of approach that utilize the L_{ca} function derive it in the same general manner, some writers have added physical definitions of L_{ca} which are not rigidly correct while others leave the physical interpretation of L_{ca} purposely vague. For example, L_{ca} has been defined as "the distance along the stream channel to a point opposite the centroid of the drainage area."⁶ This definition is actually incorrect since the location of the centroid of an area depends solely on its shape, and L_{ca} , derived from the area-distance curve, obviously depends on other factors as well. On the other hand, L_{ca} is sometimes vaguely defined as "the distance to the center of area."⁹

Physically speaking, just what is L_{ca} ? Fig. 3 shows a hypothetical drainage area. Axis x-x is drawn perpendicular to the major axis of the area through

⁹ "Applied Hydrology," by R. K. Linsley, M. A. Kohler, and J. L. H. Paulhus, McGraw-Hill, New York, 1949, p. 456.

the base of the area. The true area-centroid is then located a distance \bar{y} above this axis where

$$\bar{y} = \frac{\sum^A y_1 \Delta A}{A} \dots\dots\dots (2)$$

in which ΔA is an elemental area, y_1 is its distance from the axis $x-x$, and A is the total area. The term L_{ca} can be expressed by a similar formula

$$L_{ca} = \frac{\sum^A y_2 \Delta A}{A} \dots\dots\dots (3)$$

in Eq. 3 y_2 is the distance measured along the stream to the point on the channel at which overland runoff resulting from rainfall on ΔA first reaches the stream.

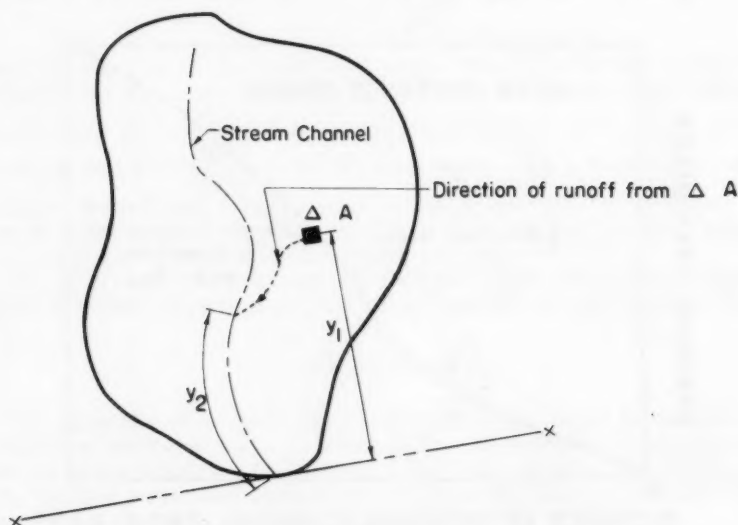


FIG. 3.—HYPOTHETICAL DRAINAGE AREA

LAND SLOPE

Defining a characteristic which adequately describes land slope has proved difficult. However, this characteristic is a major factor in indicating the travel time of overland flow, so several methods for determining land slope have been developed.

Most of the methods generally used for defining land slope take into account the general slope of the entire basin including the tributary stream network contributing to the major watercourse, but excluding the major watercourse.

Average land slope has been defined as equal to the plotted contour interval times the total length along all contours in the drainage-basin divided by the drainage area. However, measuring the length of contours proved a tedious and time-consuming operation and hence the following short-cut methods were derived and are used:

In the intersection-line method, a grid of uniformly spaced parallel and perpendicular lines is laid over a contour map of the drainage area. The number of contours crossing each subdivision of the grid within the area is counted.

The land slope in either grid direction is then $s = \frac{N \Delta Z}{L}$ where N is the total number of contour crossings for all lines in one direction, ΔZ denotes the contour interval, and L is the total length of grid lines in one direction within the area. The average land slope is then given by some relation between these two values of slope or by

$$s' = \frac{N' \Delta Z}{L'} \sec \theta \quad \dots \dots \dots (4)$$

where N' and L' are the sum in both grid directions of the values previously defined as N and L , and θ is the average angle of intersection between the contours and grid lines. Often a value of 1.57 is used for $\sec \theta$ as it is the average secant of angles from 0° to 90° . The Corps of Engineers uses this method of finding land slope.⁸

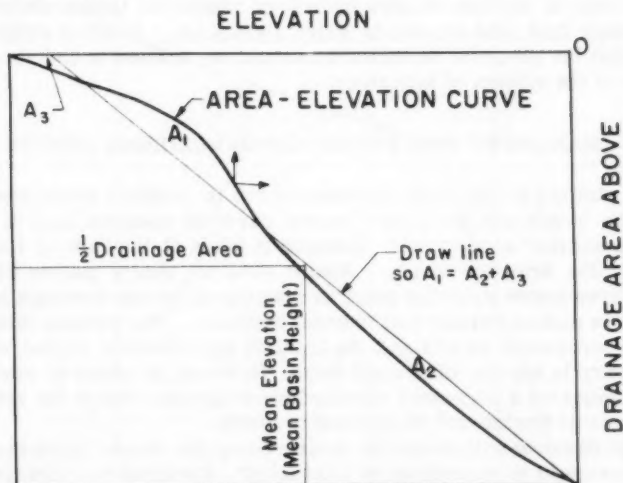


FIG. 4.—LAND SLOPE

Another method of defining land slope, or more accurately, basin slope, utilizes an area-elevation curve in which the drainage area above a certain elevation is plotted against that elevation. Area-elevation data can be assembled by planimetry of the area enclosed by each contour and the basin divide and plotted as shown on Fig. 4.

The mean basin height can be obtained by integration of the area-elevation curve and dividing the result by the drainage area. It can also be obtained by the method shown in Fig. 4 in which a line is drawn such that $A_1 = A_2 + A_3$. The intersection of this line with a line drawn parallel to the abscissa at 50% of the drainage area indicates the mean basin height. Mean basin slope is then defined as two times the mean basin height divided by the length of the channel.

Another curve can be plotted conveniently by using the same data derived for producing an area-distance curve. This is called a modified area-elevation curve. Rather than cutting the drainage-basin into slices between adjacent contours, it can be divided into sub-drainage basins contributing to equal reaches along the length of the principal drainage channel to correspond with the method used in computing the area-distance curve. The preferable method, especially for larger rivers, is to break the channel at the tributaries just above the point where they join the main channel as described under the heading "Basin Shape." In this case, area no longer refers to area lying above a certain elevation but rather to the area contributing runoff to the major stream above the point at which the stream bed is at a certain elevation. Thus the mean basin height is obtained actually weighted by distribution of drainage area long the stream profile. Mean elevation and mean basin slope are correspondingly weighted.

The foregoing method saves considerable computation time over the usual method for computing an area-elevation curve by utilizing one set of area data as the basis for both curves. In the opinion of the writers this results in a more meaningful mean basin height and mean basin slope. It is undoubtedly not applicable in studies of snow hydrology where the temperature-elevation curve makes true land elevations more significant. Further study may also indicate that the proposed modification should not be used in certain other subdivisions of the subject of hydrology.

SUGGESTED PROCEDURE FOR DEVELOPING CURVES

In the plotting of the area-elevation curve to compute mean elevation and mean basin height and the area-distance curve to compute L_{ca} , it makes no difference whether zero channel distance is taken at the base of the drainage area or at the drainage divide. Also it does not really matter whether the drainage area above a certain point on the channel or the drainage area below that point is plotted versus that channel distance. The plotting units may be actual or percentage values and the location and sense of scales could conceivably vary in any one of several ways. However, in order to eliminate any possible confusion a suggested standard procedure similar to the method used by the Corps of Engineers⁸ is outlined herewith:

Channel distance will always be measured up the channel from the base and will be expressed in percentage of total length. Elevation will always be measured above the stream-bed elevation at the base of the drainage-basin and will be expressed as the percentage of the total elevation drop from one end of the stream to the other. Drainage area will also be expressed as a percentage of the total. It will be recalled that in the development of the area-distance relationship the term area referred to is the area of the drainage-basin contributing below the point along the stream in question. On the other hand the area-elevation relationship required the consideration of the contributory area above the point in question. This distinction need cause no confusion in actual practice, as will be shown.

The fact that all scales go from 0% to 100% enables the engineer to plot both curves on a square piece of graph paper. Since both curves appear roughly as diagonals on such a graph, the scales are oriented so that they will cross only once, thus eliminating any danger of confusion (Fig. 5). The value of L_{ca} is then found by measuring the area above the area-distance curve in square inches and converting to square percentage, for example, for horizontal and verti-

cal scales of 1 in. = 10%, the conversion factor is 1 sq in. = $(100\%)^2$. This figure divided by 100% is the L_{ca} expressed as a percentage of the total length.

To find mean basin height (mean elevation), the area above the area-elevation curve is determined, converted to square percentage and divided by 100%. The result is the mean basin height expressed as a percentage of the total height. Mean basin slope is then mean basin height divided by one-half the total channel length, both values having been converted from percentage to actual lengths.

The modified area-elevation curve is plotted in a similar fashion to that described for the standard area-elevation relationship.

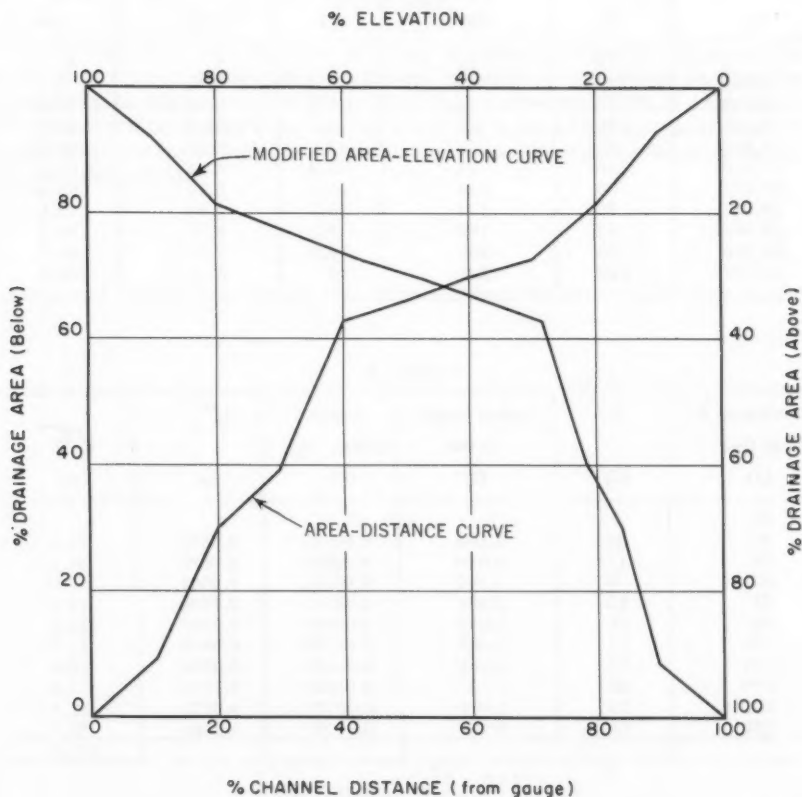


FIG. 5.—HYPOTHETICAL DRAINAGE BASIN

Example.—The necessary points for plotting a modified area-elevation curve and an area-distance curve for a hypothetical drainage-basin are shown in Table 1. Sub-divisions on the stream were made at even tenth points along the total channel length. Other methods, as previously noted, could have been used. Note that distance is measured in an upstream direction and that area and elevation are measured below the point on the channel. The modified area-elevation curve is plotted using the lower right-hand corner as the origin.

An alternate solution is to add another column to Table 1 for percentage area above, which is determined by subtracting percentage area below from 100%. The modified area-elevation curve can then be plotted directly using the origin in the upper-right corner.

TABLE 1.—HYPOTHETICAL DRAINAGE BASIN (DRAINAGE AREA = 8.10 SQ MILES, CHANNEL LENGTH TO BASIN DIVIDE = 5.42 MILES)

Length, L, in feet (1)	% L (2)	Elevation, E, in feet (3)	% E below (4)	Area, A, in square miles (5)	% A below (6)
0	0	21	0	0	0
2,860	10	43	11	0.70	8.6
5,720	20	54	16.5	2.43	30.0
8,590	30	63	21	3.14	38.8
11,450	40	78	28.5	5.07	62.6
14,310	50	94	36.5	5.28	65.2
17,170	60	116	47.5	5.62	69.4
20,030	70	134	56.5	5.84	72.1
22,900	80	180	79.5	6.55	80.8
25,760	90	202	90.5	7.51	92.7
28,620	100	221	100	8.10	100.0

TABLE 2

Elevation, E, in feet (1)	E (2)	Reach length, in feet (3)	Reach slope, S_i (4)	$\sqrt{S_i}$ (5)	$\frac{1}{\sqrt{S_i}}$ (6)
21					
43	22	2,860	0.00770	0.0878	11.4
54	11	2,960	0.00385	0.0621	16.1
63	9	2,860	0.00315	0.0561	17.8
78	15	2,860	0.00525	0.0725	13.8
94	16	2,860	0.00560	0.0748	13.4
116	22	2,860	0.00770	0.0878	11.4
134	18	2,860	0.00630	0.0794	12.6
180	46	2,860	0.01608	0.1270	7.9
202	22	2,860	0.00770	0.0878	11.4
221	21	2,860	0.00735	0.0858	11.7
					127.5

$$S_{st} \left(\frac{10}{127.5} \right)^2 = 0.00615 \text{ ft per ft}$$

A sample computation of the weighted values of mean basin height, mean elevation and land slope (mean basin slope) based on the data given in Table 1 is as follows:

Area above modified area-elevation curve = (3810%)²

Mean Basin Height = (0.381) x (221 - 21) = 76.2 ft

Mean Elevation = 21 + 76.2 = 97.2 ft (m.s.l.)

Land Slope (Mean Basin Slope) = 76.2/14,310 = 0.00532 ft per ft
= 28.1 ft per mile

The computation of L_{ca} is as follows:

$$\text{Area above area-distance curve} = (4330\%)^2$$

$$L_{ca} = (0.433) \times (5.42) = 2.34 \text{ miles}$$

Determination of the Channel Slope.—In Table 2 stream slope for the same hypothetical drainage basin considered in Table 1 is computed by the method previously reviewed in which the slope of the principal drainage channel is expressed as the slope of an equivalent stream having the same travel time and same length.^{6,8} Pertinent information from Table 1 is thus repeated in Table 2.

SUMMARY

A review of the more important physical characteristics of drainage-basins has been presented along with a suggested standard procedure of plotting the important area-distance curve, from which L_{ca} can be computed, and the area-elevation curve from which the land slope (mean basin slope) can be computed. An additional characteristic, developed from the modified area-elevation curve, has been described.

The first part of the paper deals with the history of the subject, and the second part with the present state of the art. The author discusses the various methods of treatment, and the results of the different studies. He also mentions the importance of the subject, and the need for further research. The paper is well written, and the author's conclusions are based on a thorough knowledge of the subject.

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DEVELOPMENT OF FLOW IN TANK DRAINING

By David Burgreen¹

SYNOPSIS

A theoretical study is made of the development of velocity with time when a valve at the end of a drain pipe is suddenly opened. The effect of friction, pressurization, and length of drain pipe is examined. The true velocity, obtained by considering the fluid inertia, is compared to the velocity obtained by the more well-known method which disregards the fluid inertia and assumes that the draining is governed only by the prevailing head.

INTRODUCTION

The problem treated herein is a generalization of a practical problem dealing with the shutdown of a reactor by the rapid draining of moderator or reflector liquid. It is important to know quantitatively the rate of draining of the liquid from the tank to ascertain that it is sufficiently high to give rapid reactor shutdown. The mechanics of rapid liquid-level change or of rapid tank draining falls in the realm of non-steady flow. The moderator or reflector liquid is initially at rest, and the order of magnitude of time required to reach

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peak velocity is the same order of magnitude of time, from a nuclear point of view, in which the shutdown must be completed.

The more well-known type of tank-draining problem is the one in which the rate of discharge is governed by the decreasing head in the tank. Although the rate of flow varies with time, this is not a true non-steady flow problem, but rather one of slowly varying steady flow. In flow under decreasing head the inertia forces are neglected, whereas in non-steady flow the presence of inertia forces is implied. The flow relationships that are derived without the inclusion of inertia forces are designated as "terminal flow" relationships in this paper, since they represent an approximation of the terminal phase of the flow.

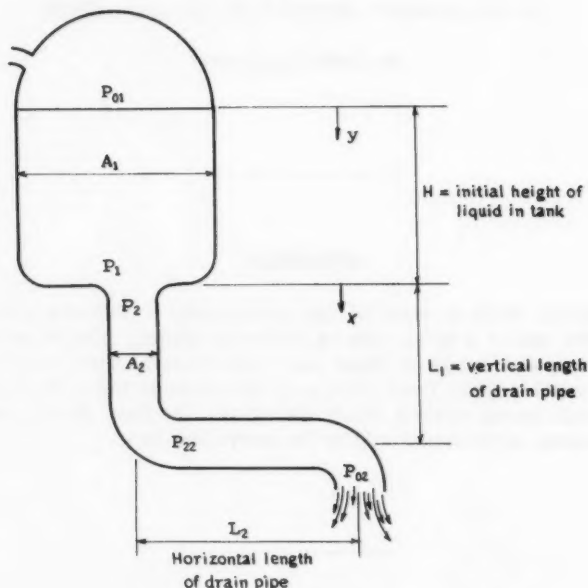


FIG. 1.—TANK AND DRAIN PIPE ARRANGEMENT

A related problem dealing with the hydrodynamics of a conservative mass and energy system has already been studied.² The present problem deals with a non-conservative mass and energy system, in which mass and energy in the form of draining liquid is continuously being lost from the system.

BASIC EQUATIONS

The system consists of a large pressurized tank from which liquid drains through a drain pipe of smaller diameter having a vertical and a horizontal length. It drains under the influence of gravity, assisted in some cases by a gas pressure on the surface of the liquid. Fig. 1 shows the arrangement of

² "Hydrodynamics of Liquid Poison Scram System," by David Burgreen, Nuclear Science and Engineering, Vol. 4, No. 1, July 1958.

tank and drain pipe. The summation of forces on the slug of liquid in the tank gives

$$P_{01} A_1 - P_1 A_1 + w (H-y) A_1 - \frac{w}{g} (H-y) A_1 \frac{d^2 y}{dt^2} = 0 \dots\dots\dots (1)$$

It is assumed in deriving Eq. 1 that friction forces retarding the flow in the tank may be neglected. A balance of forces on the slugs of liquid in the vertical and horizontal lengths of drain pipe give, respectively:

$$-P_{22} A_2 + P_2 A_2 + w A_2 L_1 - \frac{w}{g} L_1 A_2 \frac{d^2 x}{dt^2} - \frac{w}{2g} A_2 f_1 \frac{L_1}{D_2} \left(\frac{dx}{dt} \right)^2 = 0 \dots (2a)$$

and

$$P_{22} A_2 - P_{02} A_2 - \frac{w}{g} L_2 A_2 \frac{d^2 x}{dt^2} - \frac{w}{2g} A_2 f_1 \frac{L_2}{D_2} \left(\frac{dx}{dt} \right)^2 = 0 \dots\dots\dots (2b)$$

in which x represents the displacement of a fluid particle in the drain pipe and is most clearly defined by the continuity relationship $A_1 y = A_2 x$. The addition of Eqs. 2 yields

$$-P_{02} + P_2 + w L_1 - \frac{w}{g} L \frac{d^2 x}{dt^2} - \frac{w}{2g} F \left(\frac{dx}{dt} \right)^2 = 0 \dots\dots\dots (3)$$

In Eq. 3, $L = L_1 + L_2$ and F represents the number of velocity heads lost through pipe friction and would include entrance, exit, bend, and valve losses in addition to the straight pipe losses. At the junction of the pipe and tank the pressure-velocity relationship is the same as for steady flow.² It is

$$P_1 - P_2 = \frac{w}{2g} \left(\frac{dx}{dt} \right)^2 \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] \dots\dots\dots (4)$$

provided the length of the drain pipe is greater than its diameter.

The use of the continuity relationship, $A_1 y = A_2 x$, together with Eqs. 1, 3, and 4 produces:

$$\left[L + \frac{A_2}{A_1} H - \left(\frac{A_2}{A_1} \right)^2 x \right] \frac{d^2 x}{dt^2} + \frac{1}{2} \left[1 + F - \left(\frac{A_2}{A_1} \right)^2 \right] \left(\frac{dx}{dt} \right)^2 - g \left[H + \frac{P_{01}}{w} + L_1 - \frac{A_2}{A_1} x \right] = 0 \dots\dots\dots (5)$$

Eq. 5 is expressed in terms of the displacement of a particle in the drain tube. The flow could be expressed just as readily in terms of the drop of level in the tank, y . In deriving Eq. 5, P_{02} , the pressure at the end of the drain pipe has been assumed to be atmospheric (zero gage).

INITIAL ACCELERATION

At the start of flow the displacement and the velocity are zero. From Eq. 5 the initial acceleration is

$$\left(\frac{d^2 x}{dt^2} \right)_{t=0} = g \left[\frac{(n+1) H + L_1}{L + R H} \right] \dots\dots\dots (6)$$

where $R = A_2/A_1$ and $nH = P_{01}/w$. The pressurization in the tank is thus expressed as the number, n , of initial tank heads. The initial acceleration of the liquid in the tank, which is the same as the acceleration of the dropping surface of liquid is

$$\left(\frac{d^2y}{dt^2}\right)_{t=0} = Rg \left[\frac{(n+1)H + L_1}{L + RH} \right] \dots \dots \dots (7)$$

It is clear from Eq. 7 that without pressurization the initial acceleration of the liquid in a tank, to which a drain pipe is attached, can never be as high as g , since $R < 1$. In order to obtain an initial acceleration of g , the required pressurization nH is

$$nH = \frac{L_2 + (1-R)L_1}{R} \dots \dots \dots (8)$$

CROSS-SECTIONAL AREA OF A VERY SMALL DRAIN PIPE

When the flow area of the drain pipe is very small in comparison to the flow area of the tank, Eq. 5 simplifies to

$$L \frac{d^2x}{dt^2} + \frac{1+F}{2} \left(\frac{dx}{dt}\right)^2 - g[(n+1)H + L_1] = 0 \dots \dots \dots (9)$$

and represents flow under constant head with inertia effects. It is thus indicative of the initial flow behavior only, since the approximation of Eq. 9 is not valid for large x . The first integrals of Eq. 9 with respect to time and displacement are, respectively,

$$\frac{dx}{dt} = \left[\frac{2g[(n+1)H + L_1]}{1+F} \right]^{1/2} \tanh \left[\{2g[(n+1)H + L_1](1+F)\}^{1/2} \frac{t}{2L} \right] \dots (10a)$$

and

$$\frac{dx}{dt} = \left[\frac{2g[(n+1)H + L_1]}{1+F} \right]^{1/2} \left[1 - e^{-[(1+F)/L]x} \right]^{1/2} \dots \dots \dots (10b)$$

and the displacement may be written explicitly as a function of time by combining Eqs. 10:

$$x = \frac{L}{1+F} \ln \left[1 - \tanh^2 \left(\{2g[(n+1)H + L_1](1+F)\}^{1/2} \frac{t}{2L} \right) \right]^{-1} \dots \dots \dots (11)$$

If there were a horizontal drain pipe only and no pressurization or friction then Eq. 10a would become

$$\frac{dx}{dt} = [2gH]^{1/2} \tanh \left[(2gH)^{1/2} \frac{t}{2L_2} \right] \dots \dots \dots (12)$$

Eq. 12 has been derived³ from the Bernoulli equation for non-steady flow.

From Eq. 10b it is seen that for small values of x/L the velocity in the drain pipe may be approximated by

$$\frac{dx}{dt} \approx \left[\frac{2gx[(n+1)H + L_1]}{L} \right]^{1/2} \dots \dots \dots (13)$$

³ "Fundamentals of Hydro and Aeromechanics," by Prandtl-Tietjens, McGraw Hill,

This result is compatible with Eqs. 6 and 7 and is the velocity associated with a constant initial acceleration. When the drain pipe area is extremely small, the change in head is very slow and the starting velocity of the terminal flow is not sensitive to fluid movement in the lower drain pipe, and can be expressed as

$$\left(\frac{dx}{dt}\right)_{R \rightarrow 0} = \left[\frac{2g[(n+1)H+L_1]}{1+F} \right]^{1/2} \dots\dots\dots (14)$$

TERMINAL FLOW

The solution of Eq. 5 with the inertia term $d^2x/dt^2 = 0$ is the solution of the well-known tank-draining problem in which the discharge is governed by the momentary head and the inertia of the fluid to a change in velocity is disregarded. It is of considerable practical importance and, as will be shown, it is a very close approximation of the flow for the case when both the area ratio is small and the length of drain pipe is small. Under zero inertia conditions, the solution of Eq. 5 in terms of the y -coordinate, or liquid level in the tank, is

$$\frac{dy}{dt} = R \left[\frac{2g[(n+1)H+L_1-y]}{1+F-R^2} \right]^{1/2} \dots\dots\dots (15)$$

The integral of Eq. 15 gives the time required for a specified drop in liquid level. It is

$$t = \frac{1}{R} \left[\frac{2}{g} (1+F-R^2) \right]^{1/2} \left\{ [(n+1)H+L_1-y_0]^{1/2} - [(n+1)H+L_1-y]^{1/2} \right\} \dots (16)$$

where y_0 is the liquid level at $t = 0$.

GENERAL SOLUTION

Although Eq. 5 is a non-linear differential equation with variable coefficients, the first integral, in terms of the tank level y , is readily obtained. It gives the velocity as

$$\begin{aligned} \frac{dy}{dt} = & \left\{ \frac{2gR^2}{1+F-R^2} \left[(n+1)H+L_1 + \frac{R(L+RH)}{1+F-2R^2} \right] \left[1 - \left(1 - \frac{Ry}{L+RH} \right)^{(1+F-R^2)/R^2} \right] \right. \\ & \left. - \frac{2gR^2y}{1+F-2R^2} \right\}^{1/2} \dots\dots\dots (17) \end{aligned}$$

Eq. 17 has four parameters in addition to the independent variable y/H . The effect of these parameters will be examined separately. Consider first the solution when the drain pipe lengths $L_1 = L_2 = H$, when there is no pressurization, and the friction forces are neglected. Eq. 17 becomes

$$\frac{V_y^2}{2gH} = \frac{R^2(2+2R-3R^2)}{(1-R^2)(1-2R^2)} \left[1 - \left(1 - \frac{y}{H} \frac{R}{2+R} \right)^{(1/R^2)-1} \right] - \frac{y}{H} \frac{R^2}{1-2R^2} \dots\dots\dots (18)$$

Eq. 18 is plotted in Fig. 2 for several ratios of drain pipe area to tank area. When this ratio is unity, the configuration represents a pipe elbow of uniform

diameter whose vertical length is twice the horizontal length. Eq. 18 then more simply expressed as

$$\frac{V^2}{2gH} = \frac{y}{H} + \ln \left(1 - \frac{y}{3H} \right), \quad y < 2H \dots \dots \dots (19)$$

A more general expression derived from Eq. 17, neglecting friction and pressurization, for the discharge of fluid from a pipe elbow of uniform diameter of total length $L + H$ and horizontal length $L_2 = L - L_1$ is

$$\frac{V^2}{2gH} = \frac{y}{H} + \frac{L_2}{H} \ln \left(1 - \frac{y}{H} \frac{H}{L+H} \right), \quad y < H + L_1 \dots \dots \dots (20)$$

In Fig. 2, a free-fall curve, which is included for comparative purposes, shows the velocity of a free-falling body, dropped from a height H , at any ele-

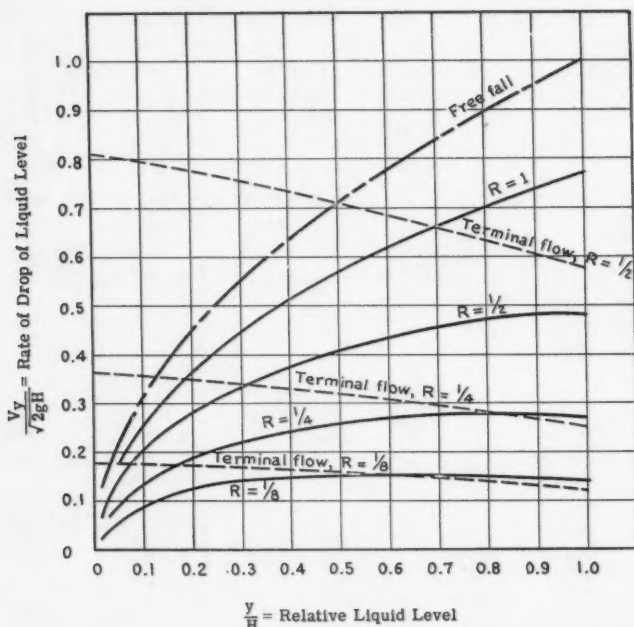


FIG. 2.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L_1 = L_2 = H$; $n = 0$; $F = 0$.

vation. It is a special case of Eq. 20, or the $R = 1$ curve when L_2 , the horizontal length of pipe, is equal to zero. The curves in Fig. 2 show the rate of fall of the liquid surface in the tank to decrease with decreasing area ratios. The range in which the flow is governed primarily by inertia forces also becomes smaller with decreasing area ratio. This may be observed through the increasingly early blending of the true flow curves with the terminal flow curves as the area ratio becomes smaller. The dotted terminal flow curves are plotted from Eq. 15 which neglects inertia effects. These are fair approximations of the terminal stages of the flow.

When the area ratio is quite large as, for example, when $R = 1/2$ the velocity-displacement curve shows that the inertia forces primarily determine the character of the flow. For large area ratios the velocity-displacement curves resemble the free-fall curve and any approximation with terminal flow would not be valid. Note that the true velocity-displacement curves indicate, properly, that the velocity starts at zero and builds up, while the terminal flow curves indicate, incorrectly, that at the start of flow the velocity is $V = (2gH)^{1/2}$.

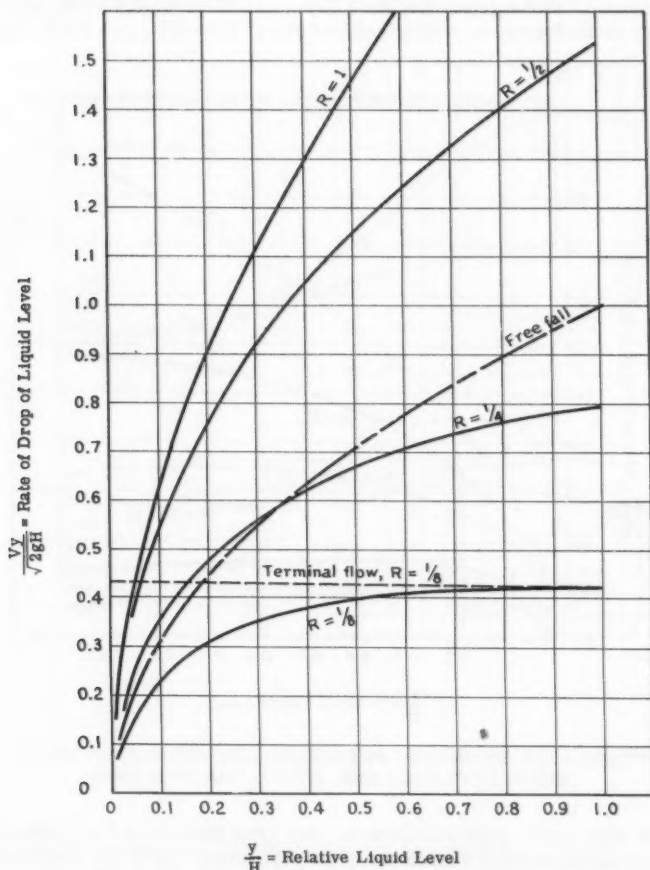


FIG. 3.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK; PRESSURIZATION: $n=10$; $F=0$; $L_1=L_2=H$.

The effect of pressurization is shown in Fig. 3. The pressure on surface of the liquid is assumed to be constant and is arbitrarily taken as ten tank heads; that is, $n = 10$. These curves show that a rate of drop in liquid level, in excess of the free-fall rate is easily obtained. It will be observed that for the selected pressurization, an area ratio of $1/4$ produces a rate of drop in

liquid level approximating free fall. The terminal flow approximation is now rather poor even for small area ratios.

The result of taking friction into account is demonstrated in Fig. 4. The total friction loss in the system is arbitrarily taken at three velocity heads (drain-pipe velocity). Comparing the curves of Fig. 4 with those of Figs. 2 and 3 it will be noted that the velocities, for corresponding area ratios, are smaller and that terminal flow is established sooner. Although terminal flow is now a better approximation to true flow, it is obviously not a satisfactory approximation, for moderate area ratios, of the early stage flow.

The effect of drain pipe length on the development of flow is well demonstrated by considering a configuration having a small area ratio typical of a

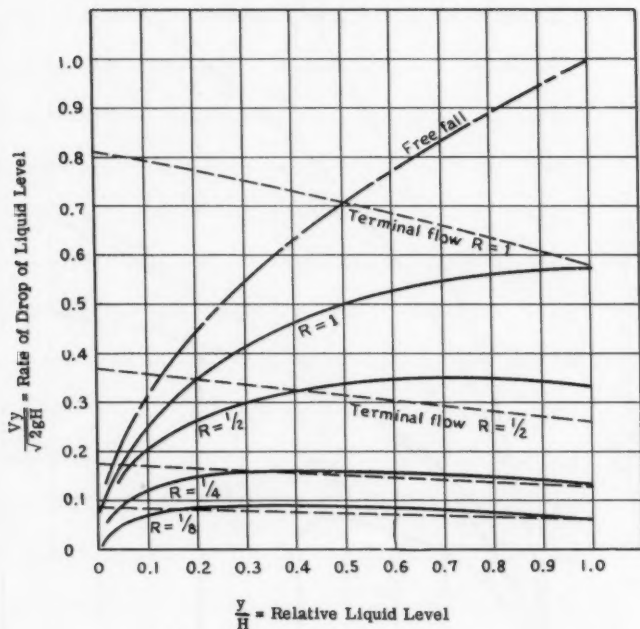


FIG. 4.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L_1 = L_2 = H$; $n = 0$; $F = 3$.

water tank with drain pipe attachment. An area ratio of 0.01 is selected, and the velocity-displacement relationship is plotted in Fig. 5 for lengths of drain pipe of zero, H , and $5H$. The curves show, in common, that the inertia transient is completed and terminal flow started when the maximum rate of flow, more or less, has been attained.

The curve in Fig. 5a represents the development of flow when a very long drain pipe, equal to $5H$, is attached to the tank. The peak velocity is not attained until about half the tank has been drained. This indicates that a long drain pipe produces a long inertia transient, and that computations of discharge based on the prevailing head lead to incorrect results — even when the

area ratio is small. When the drain pipe is of moderate length, equal to H , then the peak velocity is reached when one-tenth the tank has been drained. This is shown in Fig. 5b.

The fundamental differences between inertia flow and terminal flow are brought out by the curves in Fig. 6 in which the fluid acceleration is plotted against the drop in liquid level. Since terminal flow is flow without acceleration, the deviation from terminal flow is measured by the magnitude of the acceleration. The parameters are the same as those used in the velocity-displacement curves of Fig. 2; namely, unpressurized frictionless flow with $L_1 = L_2 = H$. The uppermost curve is a straight horizontal line representing

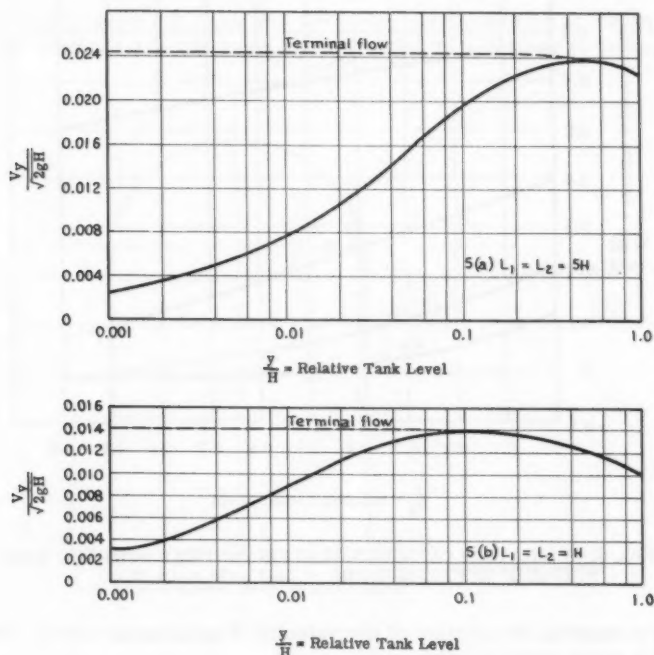


FIG. 5.—DEVELOPMENT OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $R = 1/100$; $n = 0$; $F = 0$.

free fall, or the equation $(1/g)(d^2y/dt^2) = 1$. An examination of the curves will show that for each case the acceleration is a maximum at the start of flow and thereafter decreases continuously with increasing displacement. As the area ratio becomes small the curves show a range of decelerating flow which corresponds to the region of decreasing velocity. Terminal flow is represented by the abscissa, $(1/g)(d^2y/dt^2) = 0$. The general trend of the curves is toward zero acceleration as the area ratio approaches zero. This general trend will prevail also for configurations other than $L_1 = L_2 = H$, but with longer drain pipes will tend toward zero acceleration at a slower rate.

To obtain time-displacement curves it is necessary to integrate Eq. 17. Analytically, it appears to be possible to perform the integration only for specific values of F and R . In general the time-displacement curve would be ob-

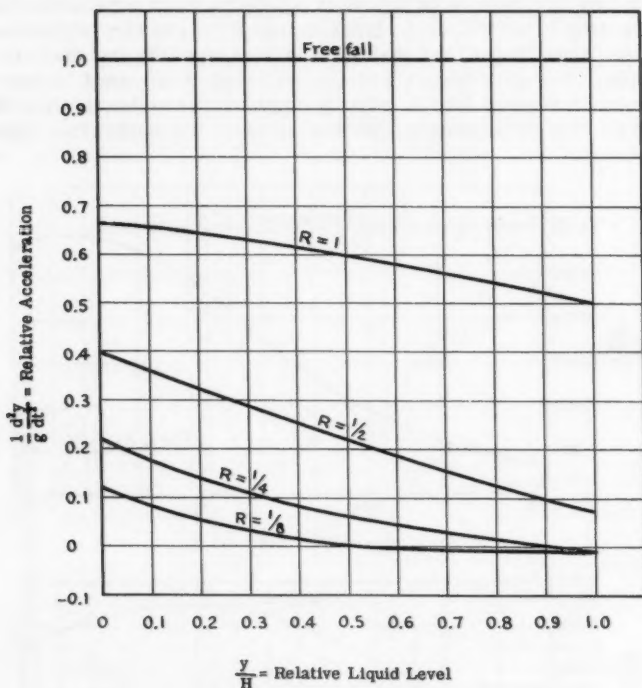


FIG. 6.—VARIATION OF ACCELERATION OF FREE SURFACE WITH LIQUID LEVEL IN TANK: $L_1 = L_2 = H$; $n = 0$; $F = 0$.

tained by numerical integration of the velocity-displacement curve. The time to reach a given level, y_n , is

$$t_n = n \Delta y \sum_{i=1}^{i=n} \frac{1}{V_i} \quad \dots \dots \dots (21)$$

Initially $1/V$ tends to infinity. This represents a difficulty for numerical integration. For small values of y/H Eq. 17 becomes

$$V_y^2 \approx \frac{2 g y R [(n+1)H + L_1]}{L + R H}, \quad \frac{y}{H} \rightarrow 0 \quad \dots \dots \dots (22)$$

An examination of Eqs. 6 and 7 indicates that Eq. 22 is simply

$$V_y^2 \approx 2 \left(\frac{d^2 y}{dt^2} \right) y, \quad t \rightarrow 0 \quad \dots \dots \dots (23)$$

or the expression for velocity when the acceleration is constant. This approximation may be used to start the integration, since for constant acceleration

$$t \approx \left[\left(\frac{2y}{\frac{d^2y}{dt^2}} \right)^{1/2} \right] \dots \dots \dots (24)$$

for small values of t

DISCHARGE THROUGH A WEIR ANNULUS

It is fairly apparent from the foregoing material that to obtain the rapid drainage required for the rapid shutting down of a reactor it is necessary to

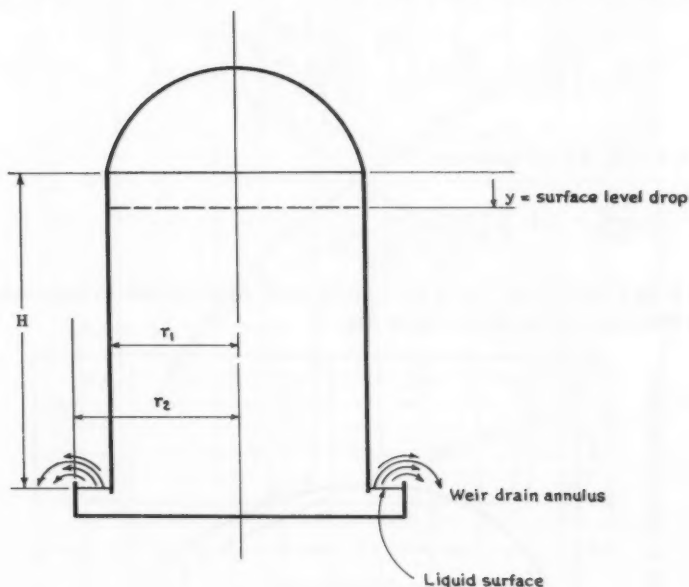


FIG. 7.—CONFIGURATION OF WEIR DRAIN SYSTEM

have a large discharge area. Rather than use a very large pipe or group of pipes, it is expedient to use an annular weir discharge as shown in Fig. 7. In this arrangement the length of the drain pipe $L = L_1 + L_2$ is assumed very small in comparison to $(A_2/A_1)H$. The solution of Eq. 5, disregarding pressurization and friction, is

$$\frac{V^2 y}{2 g H} = \frac{R^2}{1-2 R^2} \left[1 - \frac{y}{H} - \left(1 - \frac{y}{H} \right)^{(1/R^2)-1} \right] \dots \dots \dots (25)$$

Eq. 25 is plotted in Fig. 8 for ratios of drain pipe area to tank area of 1/2, 1/4, 1/8. The corresponding terminal-flow curves are also shown. Note that

now, in the absence of any length of drain pipe, terminal flow is a better approximation of the true flow, and for a practical application, would give a good approximation of the time required to completely drain a tank. It is, however, not satisfactory when the interest is primarily in the starting flow, which it does not approximate very well.

Eq. 25 can be integrated to give the drain time explicitly. When $R < 1/\sqrt{2}$ the drain time is

$$T = \left(\frac{H}{2g} \frac{R^2}{1 - R^2} \right)^{1/2} \frac{\Gamma\left(\frac{R^2}{2 - 4R^2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1 - R^2}{2}\right)} \dots \dots \dots (26a)$$

in which Γ is the gamma function. When $R > 1/\sqrt{2}$

$$T = \left(\frac{H}{2g} \frac{R^2}{2R^2 - 1} \right)^{1/2} \frac{\left(\frac{1 - 3R^2}{2 - 4R^2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{2 - 5R^2}{2 - 4R^2}\right)} \dots \dots \dots (26b)$$

For $R = 1/\sqrt{2}$, Eq. 25 becomes

$$\frac{V^2 y}{2gH} = \left(1 - \frac{y}{H}\right) \ln \left(\frac{1}{1 - \frac{y}{H}} \right) \dots \dots \dots (27)$$

As $1/\sqrt{2}$ is a very large area ratio, it is well represented in the early stage flow by free-fall. This is shown in Fig. 9.

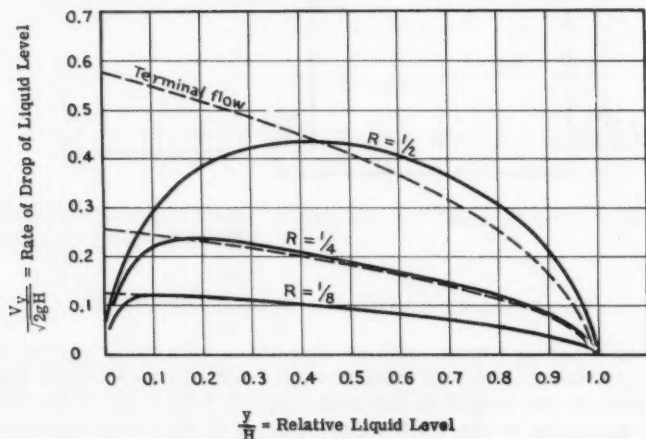


FIG. 8.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L < (A_2/A_1)H$; $n=0$; $F=0$.

The time required for the complete draining of the tank when $R = 1/\sqrt{2}$ is obtained by integrating Eq. 27 or from Eqs. 26. It is

$$T = \left(\frac{\pi H}{g} \right)^{1/2} \dots \dots \dots (28)$$

Fig. 10 shows the elevation of the liquid surface at any time. It is plotted for $R = 1/\sqrt{2}$, for free-fall, and for (improperly assumed) terminal flow. Note that until about three tenths of the tank has been drained the time-displacement relationship is practically that of free-fall. The divergence becomes more significant as the draining of the tank nears completion. To complete the draining, 25% more time is required than the time it takes for a body to fall freely through this distance. Terminal flow is not at all representative of the true state of affairs. The fact that the free-fall and terminal flow curve meet at $y/H = 1$ is fortuitous.

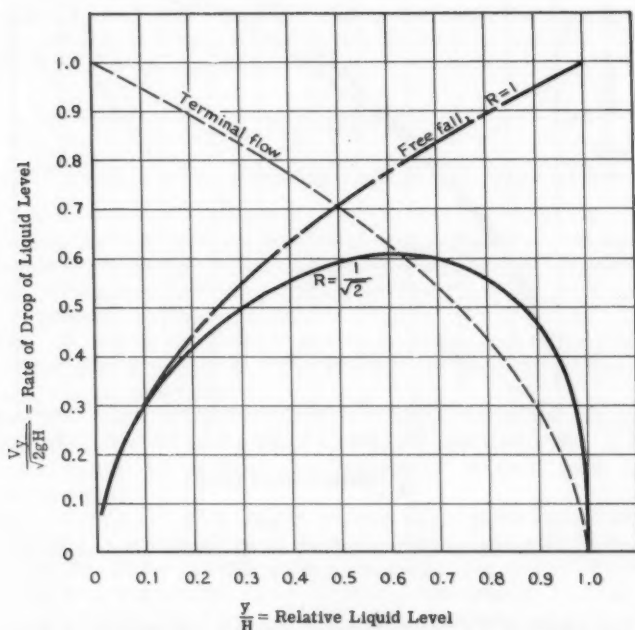


FIG. 9.—VARIATION OF FREE SURFACE VELOCITY WITH LIQUID LEVEL IN TANK. $L \ll (A_2/A_1)H$; $n = 0$; $F = 0$; $R = 1/\sqrt{2}$.

CAVITATION

At the uppermost point of the drain pipe, near the junction with the tank, the pressure in the liquid may be quite low at some stage of the flow. If the pressure at this point is less than the vapor pressure of the liquid, then flow separation or cavitation may occur. The pressure at this point is obtained from Eq. 3. It is

$$\frac{P_2}{w} = \frac{P_{02}}{w} - L_1 + \frac{L}{g} \frac{d^2x}{dt^2} + \frac{F}{2g} \left(\frac{dx}{dt} \right)^2 \dots \dots \dots (29)$$

In Eq. 29, P_{02} is atmospheric pressure and P_2 is the absolute pressure. If P_{cr} is the vapor pressure of the liquid then $(P_{02}/w) - (P_{cr}/w) = h_{cr}$ would

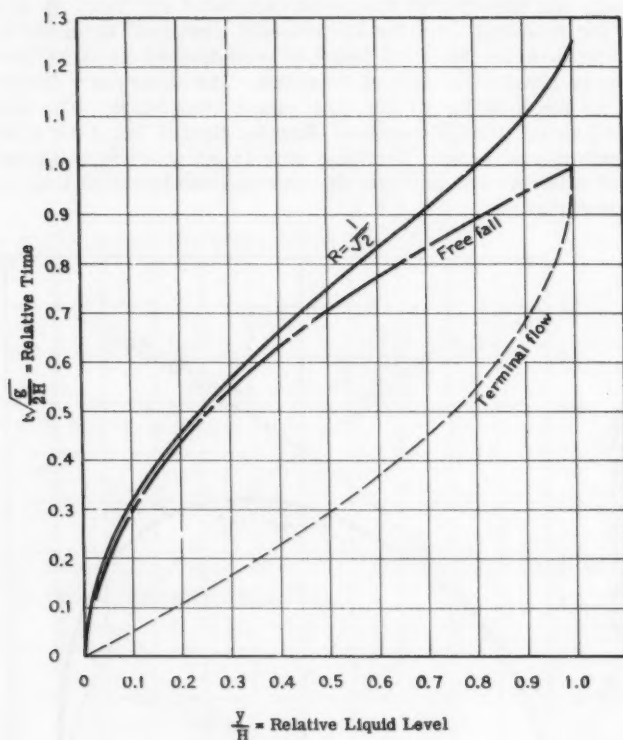


FIG. 10.—CHANGE IN LIQUID LEVEL WITH TIME. $L \ll (A_2/A_1)H$; $n=0$; $F=0$; $R=1/\sqrt{2}$.

represent the height of a column of liquid that can be supported when separation is about to occur. Separation will not occur as long as $(P_{cr}/w) < (P_2/w)$ or

$$\frac{P_{cr}}{w} < \frac{P_{02}}{w} - L_1 + \frac{L}{g} \frac{d^2x}{dt^2} + \frac{F}{2g} \left(\frac{dx}{dt} \right)^2 \quad \dots \quad (30)$$

and separation will occur when

$$L_1 - \frac{L}{g} \frac{d^2x}{dt^2} - \frac{F}{2g} \left(\frac{dx}{dt} \right)^2 > h_{cr} \quad \dots \quad (31)$$

Thus when the fluid is stationary and not accelerating ($d^2x/dt^2 = dx/dt = 0$) separation occurs when the vertical leg L_1 exceeds h_{cr} . At the start of flow,

when $dx/dt = 0$, in order for separation to occur, it would be necessary that

$$\frac{L}{g} \frac{d^2x}{dt^2} < L_1 - h_{cr} \dots\dots\dots (32)$$

At any given time or at any liquid-level displacement there will be an associated acceleration and velocity. When the acceleration and velocity fail to satisfy the inequality, Eq. 31, it will demonstrate that no separation has taken place. When separation takes place the draining of the tank is no longer governed by Eq. 5.

Before the pressure at any point in the pipe reaches the absolute vapor pressure of the fluid and produces flow separation, it will pass through a pressure range less than atmospheric. There would then be a tendency for air to back up through the pipes. However, as in the case of a siphon, as long as there is a substantial flow velocity in the drain pipe the air will not move counter the flow and the pipe will flow full.

SUMMARY

The major findings of the foregoing study of tank draining are the following:

1. When the ratio of drain pipe area to tank cross-sectional area is small and the length of drain pipe is short, then the conventional manner of computing the rate of draining, which disregards the inertia of the fluid, may be used to obtain reasonably accurate results.
2. When the area of the drain pipe is a large fraction of the tank area, the rate of draining is given by Eq. 17. The neglect of the fluid inertia, in this case, leads to incorrect results.
3. When the length of the drain pipe is a large fraction of, or greater than the initial height of fluid in the tank, it is necessary to use Eq. 17 to compute the rate of draining, and neglect of the inertia of the fluid, here again, leads to incorrect results.
4. Since either large area ratios or long drain pipes have a strong influence on the flow, a combination of moderate area ratio and drain pipe length also requires Eq. 17 for the proper computation of the rate of draining.
5. As one may expect, the effect of drain-pipe friction is to decrease the rate of draining while the effect of tank pressurization is to increase the rate of draining. For expediency the form of the friction term was taken as the steady state form, although the flow is actually non-steady.

APPENDIX.—DERIVATION OF MOMENTUM EQUATION BY ENERGY CONSIDERATIONS

The kinetic energy of the liquid in the tank is

$$\text{K.E.} = \frac{w A_1 (H-y) \dot{y}^2}{2g} \dots\dots\dots (33)$$

and in the drain pipe it is

$$K.E. = \frac{w A_2 (L_1 + L_2) \dot{x}^2}{2 g} = \frac{w A_2 L \dot{x}^2}{2 g} \dots \dots \dots (34)$$

The potential energy, using $x = 0$ as a datum is

$$P.E. = \frac{w}{2} A_1 (H-y)^2 \dots \dots \dots (35)$$

for the tank liquid and

$$P.E. = -\frac{w}{2} A_2 L_1^2 - w A_2 L_1 L_2 \dots \dots \dots (36)$$

for the drain pipe. The rate at which kinetic and potential energy is being lost from the system, in the form of draining liquid is

$$\dot{x} \left(-\frac{w A_2 \dot{x}^2}{2 g} + \frac{w}{g} A_2 L_1 \right)$$

and is equal to the rate of change with time of the kinetic and potential energy as given by Eqs. 33 through 36:

$$\begin{aligned} \frac{d}{dt} (K.E. + P.E.) &= \frac{d}{dt} \left[\frac{w}{2 g} A_1 \left(H - \frac{A_2}{A_1} x \right) \left(\frac{A_2}{A_1} \right)^2 \dot{x}^2 \right. \\ &\quad \left. + \frac{w A_2}{2 g} L \dot{x}^2 + w A_1 \left(H - \frac{A_2}{A_1} x \right)^2 - \frac{w A_2 L_1^2}{2} - w A_2 L_1 L_2 \right] \\ &= -\frac{w A_2 \dot{x}^3}{2 g} + w A_2 L_1 \dot{x} \dots \dots \dots (37) \end{aligned}$$

Eq. 37 yields

$$\left[L + \frac{A_2}{A_1} H - \left(\frac{A_2}{A_1} \right)^2 x \right] \dot{x} + \frac{1}{2} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] \dot{x}^2 - g \left[H + L_1 - \frac{A_2}{A_1} x \right] = 0 \dots (38)$$

Eq. 38 is identical to Eq. 5 with $n H = F = 0$, which was derived from momentum considerations.

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PROBLEMS CONCERNING USE OF LOW HEAD RADIAL GATES^a

Discussion by Robert G. Cox and Arthur Toch
Closure by Thomas J. Rhone

ROBERT G. COX,¹ M. ASCE.—The author is to be complimented for his excellent paper covering the problems involved in the design and operation of low head radial gates. The writer is particularly interested in the discussion on discharge characteristics.

The fourth method presented by Mr. Rhone appears to be identical to the procedure, used by the U.S. Army Engineer Waterways Experiment Station, to develop a discharge coefficient chart for radial gates on curved spillway crests. This chart was prepared for and included in "Hydraulic Design Criteria" sponsored by the Office, Chf. of Engineers, Dept. of the Army.² The WES chart (Fig. 1 herewith) and Fig. 6 of the author's paper are quite similar, except for omission of the data points on Fig. 6.

The boundary geometry of flow under a radial gate on a curved crest is complex. To simplify discharge computations many engineers have used gate openings and heads based on the gate seat and crest elevations. Furthermore, in most instances, the effects of the direction of the approaching streamlines have been ignored. Consequently, the results are often misleading when applied to other structures of dissimilar geometry.

The WES method involves the net gate opening between the gate lip and the curved crest. It also includes the head to the center of the gate opening assuming theoretical orifice discharge and atmospheric pressure under the issuing jet. Discharge coefficients were computed and plotted as a function of the angle formed by the tangents to the gate-lip and to the curved crest at the net gate opening. It is believed that this angle controls the direction of the streamlines in the immediate vicinity of the gate opening. The results should therefore be applicable to geometrically similar structures.

The WES chart was developed using model and prototype data. The data points shown are for spillway crests shaped to the form of the lower nappe of a free jet over a sharp crested weir. The effects of gate opening-head ratios on the discharge coefficients were studied. No well defined families of curves could be drawn. Therefore, an average curve was accepted for design purposes.

Mr. Rhone's Table 1 indicates unusually good agreement between the measured prototype and model discharges. The tabulated discharge computed by

^a February, 1959, by Thomas J. Rhone.

¹ Chf., Analysis Sect., Hydr. Analysis Branch, U.S. Army Engr. Waterways Experiment Sta., Vicksburg, Miss.

² "Analysis of Hydraulic Data and Development of Design Criteria," Civil Works Investigation 804.

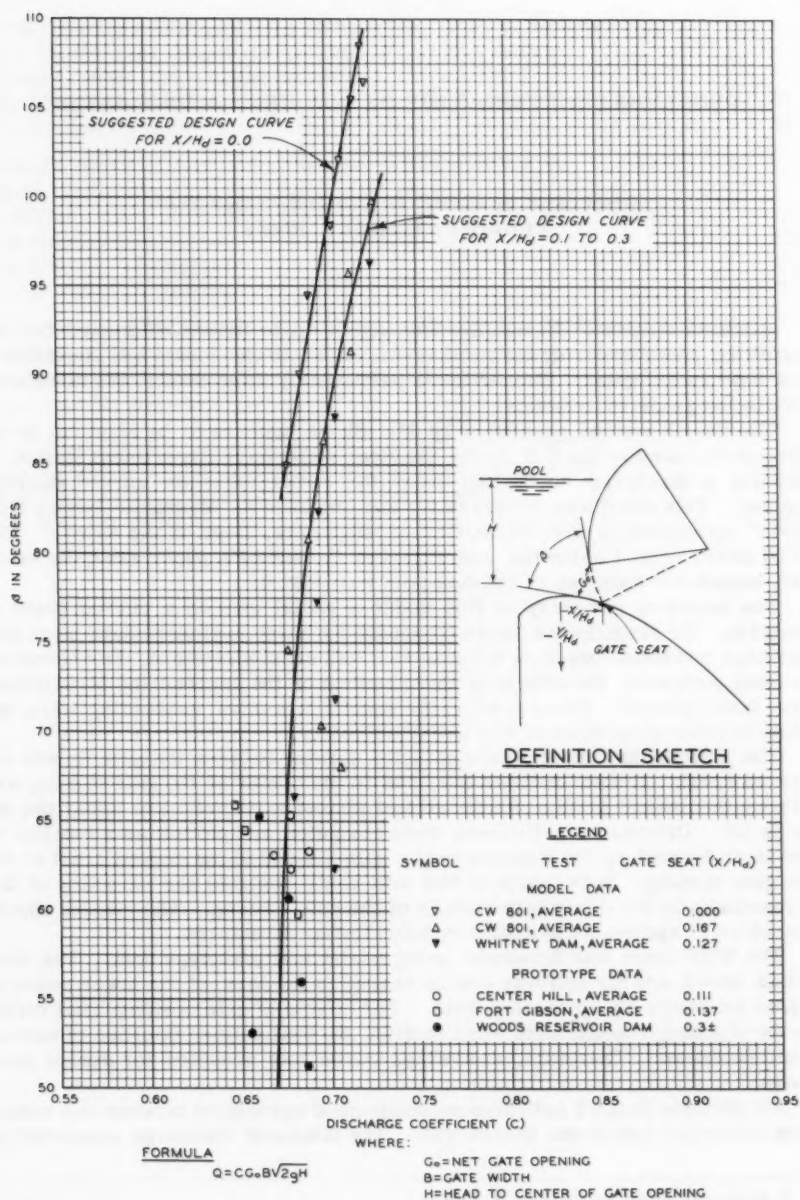


FIG. 1.—DISCHARGE COEFFICIENTS FOR RADIAL GATES.

the fourth method for the 3-ft gate opening is 4,880 cfs which is about 11% greater than the measured prototype discharge of 4,400 cfs. The writer, in an attempt to discover the reason for this difference, computed a discharge of 4,482 cfs. The latter figure is within 2% of the measured discharge. A net gate opening of 2.90 ft, an effective head of 30.75 ft, and a β of 73.1° were determined graphically from available data.³ A discharge coefficient of 0.681 from Fig. 1 was used in the computation. The writer would be interested in seeing comparative figures used by the author for his computation. Similar computations for the 6-ft and 10-ft gate openings were in close agreement with discharges computed by Mr. Rhone which are about 5% greater than the measured prototype discharges. This difference may result from the fact that Canyon Ferry crest-shape, downstream from the gate seat, is considerably flatter than the WES shape.

In the study of model data, it was noted that the discharge coefficients increased rapidly for gate openings less than 0.1 ft model. For similar prototype gate openings the coefficients were essentially constant or showed a slight decrease. This difference between model and prototype data is attributed to inherent difficulties in accurately measuring small gate openings and flows in models. Therefore, only model data with gate openings greater than 0.1 ft were used in preparation of Fig. 1.

The writer concurs with Mr. Rhone's comments concerning the need for isolating the effects of such factors as approach depth, pier shape, gate radius, trunnion position, etc. However, it would also be necessary to determine the effects of numerous combinations of these variables. A less elaborate approach for early usable results would be to standardize some of the geometrical factors in terms of the design head.

ARTHUR TOCH,⁴ M. ASCE.—As noted previously,⁵ it is believed that a general mathematical analysis for the radial gate in a flat-bottom, rectangular section is not practical. Furthermore, it is felt that continued use of Horton's article (3) as a reference in the literature on Tainter gates is regrettable. Horton not only chose to equate a contraction coefficient to a discharge coefficient but did so for gate shapes of quite dissimilar geometries.

Of the four methods presented by Mr. Rhone, only the last appears to differ from the others. The first two formulas are evidently the same, as the constant value $(2/3)\sqrt{2g}$ can be incorporated in the discharge coefficient without any loss of generality for engineering applications. Both C 's must be expected, therefore, to vary in like manner, if the geometrical characteristics of the efflux boundaries are comparable.

The third formula differs from the first two only in the inclusion of the term $1/\sin \theta$. It seems questionable to assume that inclusion of the lip angle in this manner is of significance. Also, it is apparent that Fig. 5 merely relates the cosecant and cosine of any given angle.

Because of the algebraic difference between the first three formulas and the fourth, the coefficient of discharge, for the latter, will have a quite different numerical magnitude; it must still be dependent, however, on the same

³ "Model-Prototype Conformance of Radial Gate Discharge Capacity, Canyon Ferry Dam, Montana," U.S.B.R., Hydr. Lab. Report No. Hyd 433, March 4, 1957.

⁴ Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

⁵ "Discharge Characteristics of a Tainter Gate on a Spillway," by Z. M. Glowiak, M. S. Thesis, State Univ. of Iowa, 1955.

variables or parameters. In fact, the importance of this aspect of the problem can hardly be overemphasized: The coefficient of discharge must be expected to be a function of the geometric configuration of gate and dam crest. In this respect, then, none of the methods presented by the author appears likely to shed much light on the conditions of efflux or to increase the precision of discharge prediction.

In connection with Mr. Rhone's request for further information regarding tests on radial gates, a study⁵ made at the Iowa Institute of Hydraulic Research should be cited. Although this work did not include as many variables as the author thinks necessary or desirable, Z. M. Glowiak did discuss and arrange the results he obtained in a rational manner. Another report⁶ contains fairly complete data on flow conditions and pressure distributions obtained on a model of a radial gate on a relatively high dam.

THOMAS J. RHONE,⁷ M. ASCE.—Messrs. Pariset and Michel have brought out some interesting facts in their discussion, and their extension of the coefficient-of-discharge curves is, indeed, a valuable addition. None of the crest profiles used in obtaining the curves presented in the writer's paper was as thin as the Edgard de Souza Dam crest profile; in fact, the highest free-flow coefficient was only 3.84. The larger values of the controlled flow coefficients, and the fact that for a given H the coefficient becomes essentially constant when θ is greater than 70° , rather than increasing, as shown on Fig. 6, indicates that other variables should possibly be considered in an analysis. The use of the cavitation coefficient presented is a very good measure for potential cavitation damage. A similar equation⁸ has been used as a means of determining cavitation potential in conduits and entrances, and in the case of gate slots, has been proved with model prototype comparisons.⁹ The writer assumes that the statement made by Messrs. Pariset and Michel, "To determine the maximum allowable pressure on the profile of a spillway," is in error, and should read either "minimum allowable pressure," or "maximum allowable subatmospheric pressure," as stated later.

Mr. Bowman's awareness of the problems presented in the paper and his subsequent comprehensive discussion are very gratifying. The increasing number of papers that are appearing in technical publications seem to indicate a favorable trend in "sharing the knowledge gained from basic research." It is hoped that this policy will continue, and will be as well received as in this instance.

Mr. Bowman's discussion of the various discharge quantity equations provides a clear, concise analysis of the advantages and disadvantages of each. He also provides a guide for the selection of the best combination of equations to use under various conditions of gate design, gate opening, and head above gate seal. His own method of obtaining an approximate rating curve is an excellent "short-cut" system, and is probably as accurate as any of the methods provided by the writer. The corrections for velocity of approach and con-

⁶ "Deflusso Sopra Dighe Tracimate, Somontate de Paratoio a Settore," (Flow over Dams Surmounted by Sector Gates), by V. Calderini, *L'Energia Elettrica*, Milan, Vol. XV, 1938.

⁷ Hydr. Research Engr., Div. of Engrg. Lab., Bur. of Reclamation, Denver, Colo.

⁸ "Engineering Hydraulics," edited by Hunter Rouse, John Wiley and Sons, p. 30.

⁹ "Hydraulic Characteristics of Gate Slots," by J. W. Ball, *Journal of the Hydr. Div., Proceedings, ASCE*, Volume 85, No. HY 10, October, 1953, pp. 81-114.

tractions, etc., are, for the most part, a matter of judgment, and will vary according to the practice of each design office.

The datum profiles used in the series of tests described in the paper were based on the hydraulic model investigations of the Hoover Dam spillway.¹⁰

Mr. Bowman's discussion on gate seals is very informative. His observations on prototype installations broaden the scope of the information provided in the paper, which, as he stated, was principally derived from Bureau of Reclamation practices. The writer cannot answer the question as to how the L-seal negotiates the bottom corners of a radial gate. In most of the structures investigated, the side seals were of the molded angle type, and the bottom seals were usually of the rectangular type, with a tightly fitted butt joint at the corners.

In his discussion, Mr. Cox has brought forth an unfortunate omission; namely, that the writer did not identify the source of Fig. 6. The curves were obtained from the Waterways Experiment Station chart, as he stated. The discharge quantity that he questioned was given as 4,880 cfs in the table, but should have been 4,580 cfs, which is within 4% of the measured quantity. The writer's figures in arriving at this quantity were as follows:

Net gate opening	2.96 ft
Effective head	30.77 ft
β	72.7°
Discharge coefficient	0.681

The small discrepancies between Mr. Cox's values and the writer's can easily be attributed to different interpretations of the graphical portions of the solution, and either could be accepted. Mr. Cox's explanation for the larger values for the computed discharges at bigger gate openings is entirely feasible. A steeper crest would permit higher flow quantities, other conditions being equal, than a flat crest. This is particularly true for the larger gate openings, as shown by Messrs. Pariset and Michel, and Bowman. The writer agrees with Mr. Cox's statement concerning the standardization of some of the geometrical design. The use of the design head as a basis for obtaining dimensionless ratios is successfully used in other phases of spillway design, and should be equally satisfactory in radial gate design.

Mr. Toch implies that the discharge formulas presented in the paper are also recommended by the writer; this was not the writer's intention. The four methods presented in the paper are in more or less common use, and the purpose in analyzing and discussing them was to show that each method is inadequate to some degree. The similarity between the first two formulas is in the form of the equation only. It should be obvious that the coefficient-of-discharge values in the first equation are average values obtained from numerous model studies, irrespective of gate geometry. In the second equation, the coefficients are related to the geometrical configuration of the gate. Apparently, the originator of the third method believed that the use of any gate on a crest would affect the coefficient of discharge to the extent shown in the lower graph on Fig. 5, and that the gate geometry would further modify the coefficient in the manner shown in the upper graph. The writer cannot agree that the upper graph "merely relates the cosecant and cosine of any given angle,"

¹⁰ "Studies of Crests for Overfall Dams," Boulder Canyon Projects, Final Reports, Part VI—Hydraulic Investigations, Bulletin 3.

as stated by Mr. Toch. It seems equally apparent that the graph presents a simplified method for obtaining the value of $1/\sin \theta$ by using the readily available values of the gate radius, trunnion height, and gate opening above crest. Mr. Toch is correct in believing that the coefficient of discharge is also a function of the crest shape. Previous discussions have vividly demonstrated this to be true, and it should be an important consideration in generalizing a radial gate discharge equation. The two references listed by Mr. Toch are appreciated. The thesis by Mr. Glowiak has recently been reviewed by the writer, and it is a competent piece of research work that adds considerably to the knowledge of radial gate discharge characteristics.

DISCUSSION OF RELATIONSHIPS BETWEEN PIPE RESISTANCE FORMULAS^a

Closure by Walter L. Moore

WALTER L. MOORE,¹ M. ASCE.—Several of the discussors, recognizing the inconvenience of the Colebrook-White relations for solving resistance problems, presented different approaches to aid in the achievement of a more convenient method. Emphasis was given to the limitations of the Colebrook-White transition function for unusual roughness forms and thus contributed to a more comprehensive presentation of the resistance problem. Some of the statements in the discussions indicated a misunderstanding of the paper and these will be cleared up to the extent possible.

Mr. Ackers' discussion was particularly interesting in that he described a different attempt to present the Colebrook-White relations in a more usable form than previously available. The first of the two papers to which he refers gives an excellent comprehensive review of the pipe-resistance problem. In essence, the approach described by Mr. Ackers consists of plotting the Colebrook-White relations in terms of dimensionless velocity, size, and slope parameters. It is of interest to see how the parameters he used can be formed by the combination of the more conventional dimensionless parameters of friction factor f , Reynolds Number N_R , and relative roughness e/D . Ackers' dimensionless parameters are listed below, together with their definition from his paper and their equivalent in terms of conventional parameters:

$$\begin{aligned} V &= V e / \nu = N_R \frac{e}{D} \\ R &= \frac{1}{4} \frac{D}{e} = \frac{1}{4} \frac{1}{e/D} \\ S &= \frac{2 g S e^3}{\nu^2} = f N_R^2 \left(\frac{e}{D} \right)^3 \end{aligned}$$

In Ackers' Eq. 1 the k is apparently equivalent to e as used in the rest of the paper.

The general resistance diagram, plotted in terms of the foregoing parameters, represents a logical arrangement and is based on sound dimensional reasoning. It has certain advantages which are readily apparent for problems involving a simple single pipe. A direct solution may be obtained for either the velocity, the diameter, or the head loss, if the other necessary quantities

^a March, 1959, by Walter L. Moore.

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are known. It is, thus, definitely more convenient than the Moody diagram for problems in which the diameter, the velocity, or the discharge are unknown.

The new diagram proposed by Mr. Ackers is not an improvement in all respects however. Since each of the parameters V , R , and S varies over a wide range of practical values all of the scales must be rather compressed, thus impairing the precision with which values can be read from the charts. Of course, large charts can be made to improve the precision (see charts in the pouch of Ackers' reference 7) but these become rather awkward to use and are not likely to be widely available. On the other hand, the pipe-resistance factor f varies over a rather narrow range of numerical values, and in the Moody diagram the scale for f can be chosen large enough to read with the necessary precision.

The physical significance of the variables is not as readily apparent in the Ackers' diagram as it is in the Moody diagram. The interrelationship of the velocity, size, and head loss (or slope) is apparent for the proposed diagram, but this is already apparent from the general form of the various resistance relations. The significance of changes in the roughness or the fluid viscosity are not readily apparent from the new diagram, because these variables occur in more than one of the dimensionless parameters. The Moody diagram, on the other hand, clarifies the significance of these variables and brings out more clearly the meaning of the "smooth pipe" and "rough pipe" limits.

For complex pipe systems, including transition losses or pipes in series, the writer can see little advantage in the Ackers diagram as compared with the Moody diagram. If the rate of discharge or the velocity is known, either diagram may be easily used. If the discharge is not known a trial solution will be required with either of the diagrams.

It is for these complex pipe systems, as well as for pipe networks, that the writer has found the exponential relations so useful. The writer believes it is more rational to examine the range of Reynolds Number and relative roughness likely to be encountered and to develop a suitable exponential resistance equation from the modified Moody diagram, according to the methods he has proposed, than to arbitrarily "select" an exponential formula without due consideration for the range of parameters involved in the particular problem.

Mr. Szesztay briefly reviews his work applying the Colebrook-White formula to a graphic solution of channel flow. Essentially his method consists of specifying the shape parameters of the cross section in terms of the water depth h , thus making the area and hydraulic radius a function of water depth h only. For a given sand-grain roughness k the discharge may then be expressed as a function of water depth h and slope by means of the Crump modification of the Colebrook-White formula, as given in Szesztay's Eqs. 5 and 4. The chart presented in Fig. 3 permits a ready solution of these relations. One could easily alter the relations given by Mr. Szesztay for the channel shape as a function of water depth, if he wished to use different criteria for determining the channel shape.

Although Fig. 3 is clear for the assumed k -value, it is not clear to the writer how the corresponding value of $n = 0.025$ can be stated. It would seem that the value of n corresponding to a given roughness value k would be a function of the hydraulic radius and, hence, of the water depth h . Mr. Szesztay states that statistical studies of n and k apparently fail to reveal any consistent effect of the value of the hydraulic radius R . It would seem that this result would be worthy of additional study.

In Fig. 3, the key for the slope S is listed as centimeters over centimeters whereas it appears that it should be centimeters over kilometers.

The writer was pleased to note the similarity of viewpoint presented by Mr. Miles. His extension of the concepts to a tabular comparison of e/D and Hazen and Williams' C -values was particularly valuable. The results that the Hazen-Williams C approaches 160 as the relative roughness approaches 0 was interesting. The reasonableness of this value is indicated by the C of 155 for new bituminous enameled-lined pipe of 16 in. or larger given in the 5th edition of Babbitt and Doland's "Water Supply Engineering."

The writer is grateful to Mr. Bilonok and Mr. McPherson for correcting a numerical error in the first example of the appendix. The figures shown give a value for h_f of 4.88 ft rather than 5.84 ft, as printed in the paper. The correct figure results in a value of $K_O = 0.455$ thus giving a resistance equation for this example of:

$$h_f = 0.455 Q^{1.88}$$

This numerical error also led to an incorrect value for the length of the 12-in. steel pipe appearing in Table 2. In order that the equation for the 12-in. steel pipe, $h_{f12} = 1.99 Q^{1.88}$ be correct, the length of the pipe should be 4,370 ft in place of the 3,660 ft shown in Table 2.

Mr. Bilonok also recognized that exponential formulas are in common use by practicing engineers because of their convenience, as compared with the general resistance diagram and the Darcy-Weisbach formula.

In discussing the effect of aging on pipe resistance, Mr. Bilonok states that roughness factor e , is a "complete measure of the size, shape, and distribution of individual roughness elements." Although the introduction of roughness parameter e indicates a great improvement over other methods of characterizing surface roughness, the writer agrees with Mr. McPherson and Mr. Ackers, that shape and distribution of individual roughness elements are not necessarily characterized by the value of e . For the case of statistically distributed roughness, the single parameter e appears to be a satisfactory characterization of the entire roughness. It has been shown that in the case of extreme variations of roughness form, such as grooved and corrugated pipes, some other geometric parameters are important besides a measure of the roughness heights.

The writer is puzzled by Mr. Bilonok's question regarding the finding of the exponent in example 2 where $m = 1.95$, which he says, "is not in agreement with f and e/D ." In the second example in the appendix if the values of N_R and f , or e/D listed are used to enter Fig. 1 of the paper, the value of m will be found to lie between 1.95 and 1.96.

The writer is indebted to Mr. McPherson for a very painstaking and comprehensive discussion which stressed some points which probably deserved more emphasis. Also, Mr. McPherson apparently misunderstood a number of statements in the paper.

It is obviously true that no system for approximating a complex relationship can be more precise than the relationship being approximated. Thus the proposed exponential equations are certainly no more precise (but fortunately only slightly less precise within the limitations imposed) than the Colebrook-White relationship, from which they were derived. Mr. McPherson states that

"the preciseness of the general resistance diagram implied by the 'exponential equations' of the author does not appear to be justified." It is a misunderstanding of the original paper to consider that the exponential equations imply about the preciseness of the general resistance diagram. They are merely an attempt to put the information in the general resistance diagram in a more convenient form for a number of types of resistance problems.

Mr. McPherson is concerned because the numerical values in Table 2 pass beyond the realm of water-works design practicality.

The table was designed merely as a means of illustrating the effect of a difference in the exponent m on the concept of pipe equivalents. Of course, the values extend beyond the range of water-works practice. It was not the writer's intention that the concept be limited only to water-works practice.

Mr. McPherson questions the statement that the exponential equation will agree with the general resistance diagram within 5% for a 20-fold variation in Q . This statement is significant, not because one expects to analyze a water distribution system over a 20-fold variation in Q , but in that it indicates that a satisfactory exponential equation may be developed even when the initially expected value of Q is poorly chosen. It must, of course, be realized that for the exponential approximation to apply for a full 20-fold variation, it must be developed about a central point in the range rather than about one extreme end of the range. Thus, in the first example in the appendix, which gives a head loss of 4.88 ft at a Q of 3.53 cfs, the comparison should be made at

$$Q = \frac{3.53}{\sqrt{20}} = 0.79 \text{ cfs}$$

and at $Q = 3.53 \sqrt{20} = 15.8 \text{ cfs}$.

From the equation $h_f = 0.455 Q^{1.88}$

$$h_f = 0.290 \text{ ft at } Q = 0.79 \text{ cfs}$$

$$\text{and } h_f = 82 \text{ ft at } Q = 15.8 \text{ cfs.}$$

From the general resistance diagram

$$h_f = 0.299 \text{ ft at } Q = 0.79 \text{ cfs } (f = 0.019)$$

$$\text{and } h_f = 85 \text{ ft at } Q = 15.8 \text{ cfs. } (f = 0.0135)$$

The percentage differences are as follows:

$$\text{at } Q = 0.79 \text{ cfs; } (0.292/0.299 - 1) = -2.2\%$$

$$\text{at } Q = 15.8 \text{ cfs; } (82/85 - 1) = -3.5\%$$

The writer agrees with Mr. McPherson that in designs of water-works networks a constant value of the exponent m may normally be used. The writer believes, however, that an exponent chosen from the modified Moody diagram is more rational and will also give better results than arbitrarily using the 1.85 exponent from the Hazen-Williams formula. Mr. McPherson's discussion does not seem to recognize that network problems also exist in fields other than water-works practice. In some instances it may even be desirable to use different exponents m for different branches of the system. Contrary to Mr. McPherson's statement, digital computers are not necessarily restricted to using exponent $m^{2,3}$

² "More on Pipe Line Computers," by Robert L. McIntire. Oil and Gas Journal, Vol. 57, No. 26, June 22, 1959.

Mr. McPherson has apparently not understood the writer's use of the term, "exponential resistance equations," which was defined in the paragraph immediately following Eq. 2. Thus, any equation of the form $h_f = K Q^m$ (which includes the Hazen-Williams formula as a special case) is considered an exponential formula. Mr. McPherson must have misunderstood the term in order to make the statement that he was "forced to reject as obviously unrealistic (and clearly not proven) the statement by the author: 'although the methods for analysis of networks will not be considered here, it is pertinent to note that exponential resistance relations will be most convenient for use in this type of analysis.'" Mr. McPherson goes on to propose the use of the Hazen-Williams formula for network analysis which is really a confirmation of the writer's statement although restricting it to the special case of an exponent $m = 1.85$.

Again Mr. McPherson apparently misunderstood the statement that, "thus the Hazen-Williams formula with its exponent of 1.85 corresponds to a combination of relative roughness and Reynolds number lying along the line, $m = 1.85$ on the modified Moody diagram." The discussion preceding this statement clearly deals with the effect of m on the change of head loss with change in Q . The statement should thus be taken to indicate that only along the line $m = 1.85$ will the Hazen-Williams equation have the correct exponent. Obviously the whole field of friction factors and Reynolds numbers may be covered by lines corresponding to an exponent $m = 1.85$. This is what is done in the chart presented by Mr. McPherson as Fig. D-2. The important point is that the change in f with Reynolds number occurs at the same rate for both the Hazen-Williams equation and the general resistance diagram, only along the line corresponding to $m = 1.85$. This may be clearly shown by plotting coordinates of the line $m = 1.85$ from the modified Moody diagram onto Fig. D-2. It will be seen that only along this line are the solid lines representing the Hazen-Williams equation approximately parallel to the dashed lines representing constant relative roughness on the general resistance diagram. To the right of the chart the Hazen-Williams equation would indicate a continuing decrease of friction factor with increasing Reynolds number, whereas the rough pipe relations represented on the resistance diagram show that the friction factor should become constant. It might be argued that $C_{H.W.}$ could be decreased as Reynolds number is increased, thus forcing Hazen-Williams equation to yield a constant f at high Reynolds number. This would completely destroy the added convenience of the exponential type equation in this range. Clearly what is needed in the rough-pipe range is an equation with an exponent $m = 2$ in order to fit the conditions for a constant f -value.

Further evidence of the correctness of the original statement is given by the first equation presented by Mr. McPherson under the heading, "The Hazen-Williams Formula." This equation

$$f_{H.W.} = \frac{1060}{C_{H.W.} N_R^{0.15} D^{0.015}}$$

is the same form as Eq. 5 of the paper with $m = 1.85$ because $D^{0.015}$ will be practically 1 for any reasonable value of D . Thus if C_0 is set equal to $1060/C_{H.W.}$ and $m = 1.85$, Eq. 5 will be seen to correspond to the foregoing equation.

³ "Relaxation Methods for Pipe Networks," by T. G. Chapman. Civ. Engrg. and Pub. Works Review, Vol. 51, No. 605, Nov. 1956.

The data presented by Mr. McPherson in Tables D-3 and D-4 for new linings and old linings is of interest. If the C-values given for Reynolds number and friction factor are plotted on the modified diagram, the points for the new linings, for the most part, fall between the lines of $m = 1.90$ and $m = 1.96$. For the old linings (Table D-4) the points, for the most part, lie between the lines $m = 1.94$ and $m = 1.975$. This data, also, indicates that the resistance relations for the older pipes would be better approximated by a larger value of the exponent m than for the new pipes. It is also of interest that the indicated values of m are all considerably about the 1.85 line corresponding to the Hazen-Williams equation. Thus this data also supports the writer's contention that an exponential equation with an exponent larger than 1.85, in fact one approaching 2, would be preferable for old water mains.

It is the writer's belief that the modified Moody diagram in conjunction with available data for e , or better still test results such as presented by Mr. McPherson, will enable the engineer to select the exponential equation which will be most appropriate for the type of problem with which he is dealing. The writer recognizes that there is a possibility (which at this time seems rather remote) that large changes in roughness form due to encrustation and the forming of ridges or waves might alter the form of the transition function from "smooth" to "rough" pipe conditions and thus affect the proper value of the exponent m . That the shape of the roughness should change enough to significantly affect the treatment function seems unlikely, and in the absence of reliable experimental data on this point, the modified Moody diagram appears to be a reasonable improvement over existing methods.

Additional research to explore the shape of the transition function and the geometric form of roughness for encrusted pipes would certainly be of interest. In such an investigation it may well be that the reduction in diameter will be more significant than the change in roughness since the diameter enters into the equations to the fifth power.

The work described by Dawsey attempting to relate measurements of roughness profiles to the equivalent sand grain roughness e has much appeal. To be able to run a test on a small portion of a conduit lining, or better still on a cast taken of a portion of the lining, and convert this to a hydraulic roughness parameter would be a tremendous forward step. There are many difficult problems of measurement, and meaningful analysis of the measurements, that will have to be solved before this approach is likely to yield reliable results. The magnitude of the potential gains are so great that efforts toward this goal seem well directed. The writer thanks Mr. Dawsey for noting the omission of parentheses in the Colebrook-White equation as presented in Table 1 of the paper.

The writer is grateful to all of the discussors for contributing additional information and new thoughts regarding pipe-resistance relations. The discussions have supplemented the paper by presenting other attempts to develop convenient as well as reliable methods of evaluating pipe-resistance. It is hoped that the concepts presented will lead to an improved understanding of pipe resistance relations. Time and the experience of the practicing engineer will determine which of the various approaches will prove to be most useful.

CHANNEL-SLOPE FACTOR IN FLOOD-FREQUENCY ANALYSIS^a

Closure by Manuel A. Benson

MANUEL A. BENSON,¹ M. ASCE.—Mr. Sammons is to be commended for his thorough analysis and reanalysis of the writer's data. The original article was intended to cover only the choice of a simple, yet efficient, index of main-channel slope. Mr. Sammons covered a wide scope, including basic data collection, mapping standards, flood-frequency analysis, multiple correlation, statistics, and the use of automatic computers. Much of his discussion concerns points which will be explained in detail in the pending final report of the completed project. Mr. Sammons makes some sweeping indictments of various practices, to most of which the writer wishes to plead "not guilty." For example, the writer sincerely believes he does not share that misconception of the use of automatic computers, and he is certain that he does not apply "indiscriminate weighting coefficients" to variables.

Mr. Sammons puts much stress on the distribution of the variables which are used in the regression analysis. Multiple-correlation techniques, to be valid, require linear relationships between the variables. The transformation to logarithms of the original variables was made on the basis of comparative logarithmic and rectangular plotting between the dependent and independent variables, and plottings of the residual errors at various stages against the independent variables. One variable (surface-storage area in ponds) used in the final analysis was adjusted by a "rectification constant" determined graphically to be necessary to linearize the logarithms of the variable. Finally (as suggested by Mr. Sammons) the cumulative distributions of the variables eventually used were plotted on log-probability paper and were proved to be essentially linear, within 2 standard deviations of their means. This confirmed the validity of the logarithmic transformations.

No logical reason is apparent for Mr. Sammons' statement that the curvilinear regression constant of "a" in Fig. 3, indicate that "a curvilinear logarithmic model would best fit the data." Elsewhere, Mr. Sammons refers to the "concave upward bias of the constant term a on Fig. 3." A bias is much more apt to be introduced by an a priori theory than by adherence to the data.

The distribution of most of the variables, which may influence flood peaks, is sufficiently close to log-normal, thus that distribution may be assumed. In any event, their exact distribution is not critical for the purpose of relating them to flood peaks. However, the writer is not willing to assume log-normal distribution for the annual peak discharges at a station, because this is a critical assumption. The form of the distribution becomes the principal basis

^a April, 1959, by Manuel A. Benson.

¹ Hydr. Engr. Research, U.S. Geol. Survey, Washington, D.C.

for determining extreme flood peaks. Statistical methods may be nonparametric as well as parametric. The writer prefers to correlate topographic and climatic variables with the "percentiles" of the flood-peak distribution rather than with its parameters (mean, standard deviation, and skew), simply because the use of parameters means making assumptions that are not necessarily or generally true and that may actually be in error.

Mr. Sammons cannot suppose that the investigation which he cites, in which precipitation proved to be more important than the drainage area, is generally applicable or reflects on the results for New England. If the analysis were made for a group of small drainage areas (necessarily limited in range of sizes) within a region of widely varying precipitation, it is to be expected that precipitation will prove more important. In fact, if all the drainage areas are the same size, drainage area would be of no importance whatsoever, at least as shown by the multiple-regression analysis.

The writer agrees with Mr. Sammons that the median values of variables might be better indices than the means. However, the U.S. Weather Bureau publishes only the means for climatic factors such as precipitation and temperature. The mean of topographic variables, such as elevation and land slope, is more simple to compute than the median.

The scales of topographic maps of New England were adequate to define profile elevations for the basins used in the analysis. Regions of flat slopes, less adequately mapped regions, or extremely small watersheds might require surveys or photogrammetry to determine the channel slope. The writer did not investigate the effect of variation in map scales because the available scales introduced no problems for the data studied.

The ultimate findings have related flood peaks of recurrence intervals, ranging from 1.2 to 300 yr, to the following hydrologic variables:

1. Drainage area;
2. main-channel slope;
3. surface storage in lakes and ponds;
4. precipitation intensity;
5. mean January temperature (an index of the effect of snowmelt on peaks);
and
6. orographic factor.

A report on the investigation and the findings is nearing completion.

ROLL WAVES AND SLUG FLOWS IN INCLINED OPEN CHANNELS^a

Discussion by Tojiro Ishihara, Yuichi Iwagaki and Yoshiaki Iwasa

TOJIRO ISHIHARA,¹ YUICHI IWAGAKI,² M. ASCE and YOSHIAKI IWASA.³—The writers have reviewed this paper with interest, because we also have made continuous investigations of the hydraulic characteristics of roll waves and other associated problems in laminar and turbulent flows from theoretical and experimental approaches during the past several years. Unfortunately, however, our results are quite different from the author's study, and we also do not follow his treatment of roll waves and slug flows from the knowledge we obtained through research in the hydraulics laboratory of Kyoto University.

Our research project to disclose the hydraulic characteristics of roll waves, needed by the engineering request for soil conservation, have been studied by the writers and others since 1950. Several reports were already published in Japanese⁴⁻⁷ and in English^{8,9} after completing specific problems related to roll waves. Through our research works, the basic concept of the treatment for the hydraulic characteristics of roll waves is essentially due to the studies of H. Thomas¹⁰ and R.F. Dressler.¹¹ More precisely, the derivation of the hydraulic characteristics of roll waves is based on the fact that

^a July, 1959 by Paul G. Mayer.

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⁴ "Theory of the Roll-Wave Trains in Laminar Water Flow on a Steep Slope Surface," by T. Ishihara, Y. Iwagaki, and Y. Iwasa. Studies on the Thin Sheet Flow, 5th Report., Trans. JSCE, No. 19, April, 1954, (in Japanese).

⁵ "On the Hydraulic Characteristics of the Roll-Wave Trains," by Y. Iwagaki and Y. Iwasa. Studies on the Thin Sheet Flow, 7th Report, Journal JSCE, Vol. 40, No. 1, January, 1955, (in Japanese).

⁶ "Mechanism of Water Erosion on Land-Surfaces and Roll-Wave Trains," by T. Ishihara, Y. Iwagaki and Y. Iwasa. Bulletin, Engrg. Research Inst., Kyoto Univ., Vol. 7, March, 1955, (in Japanese).

⁷ "Fundamental Studies on Land Erosions due to Rain Water Flow," by Y. Iwagaki. Dr. Engrg. Thesis, Kyoto Univ., March, 1956, (in Japanese).

⁸ "On the Rollwave-Trains Appearing in the Water Flow on a Steep Slope Surface," by T. Ishihara, Y. Iwagaki and Y. Ishihara. Memoirs, Faculty of Engrg., Kyoto Univ., Vol. 14, No. 2, March, 1952.

⁹ "The Criterion for Instability of Steady Uniform Flows in Open Channels," by Y. Iwasa. Memoirs, Faculty of Engrg., Kyoto Univ., Vol. 16, No. 4, October, 1954.

¹⁰ "The Propagation of Waves in Steep Prismatic Conduits," by H. Thomas. Proc. Hydr. Conf., Univ. of Iowa Studies in Engrg., Bulletin 20, 1940.

¹¹ "Mathematical Solution of the Problem of Roll-Waves in Inclined Open Channels," by R.F. Dressler. Comm. on Pure and Applied Math., Vol. 2, No. 2/3, 1949.

(1) the wave velocity of roll waves is definitely constant (terminal velocity by the author), regardless of their flow regime, when the final shape in wave pattern is obtained, and (2) the flow velocity is, in all cases, less than the wave velocity of roll waves in magnitude, thus the flow regime of rear side of a roll wave is shooting, whereas that of the front side, including the wave crest, is tranquil when the train of roll waves is observed from the moving coordinate system travelling at a constant speed of wave velocity. The former condition was also verified by the author, while the latter condition of Thomas was not equivalent to the author's in all points because the author divided the flow behaviors into two different wave patterns, depending on the value of the Reynolds number.

In order to ascertain our standpoint concerning the author's derivation for the hydraulic characteristics of roll waves and slug flows, our analytical treatment on the hydraulic characteristics of roll waves and the experimental verification by the direct measurement of roll waves by means of electronic devices will be first described:

Based on the principle of a constant wave velocity indicated in the foregoing, the unsteady behaviors of roll waves can be reduced to those in steady regime by eliminating the time derivatives in the basic quasi-linear partial differential equations of hyperbolic type for unsteady flows, when viewing the wave pattern from the moving origin. The hydraulic characteristics of roll waves are then easily derived in the following.

The equation of momentum for unsteady flows in open channels with a constant inclination is

$$\frac{\partial v}{\partial t} + \alpha V \frac{\partial V}{\partial X} + g \cos \theta \frac{\partial D}{\partial X} + (1 - \alpha) \frac{V}{A} \frac{\partial D}{\partial t} = g \sin \theta - \frac{g V^2}{C^2 R} \dots (1)$$

and the equation of continuity is

$$\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial X} + A \frac{\partial V}{\partial X} = 0 \dots (2)$$

in which the pressure distribution is assumed hydrostatic, x is the distance from the origin along the channel bed, t represents time, V is the mean velocity of flow, D denotes the depth of water, A is the flow area, R represents the hydraulic radius, C is Chézy's coefficient (assumed variable), g denotes the acceleration of gravity, θ is the inclination angle of channel bed, and α equals the momentum correction factor usually defined. Apparently, the velocity profile of laminar flow is of the parabolic type which was already derived by many hydraulic engineers as well as the author and the momentum correction factor becomes then a constant of 1.2 with a simple calculation. On the other hand, the evaluation of α in turbulent flow cannot be made in a definite number, because it is still a function of flow and channel characteristics, but it is satisfactorily assumed 1.05 in engineering practice. In the derivation of Eq. 1, the curvature of a stream line is neglected, and thus the influence of surface tensions is also ignored, which is different from the author's treatment. Of course, the surface tension will have to be considered for laminar flow in very steep channels.

Taking account of the preceding principle for a constant wave velocity, the time derivatives in Eqs. 1 and 2 can be eliminated by setting $\zeta = x - V_w t$,

and the resulting equations obtained by transforming the independent variables from x and t to ξ are

$$\frac{dV}{d\xi} = \frac{g(V - V_w)(dA/dD)\{\sin \theta - (V^2/C^2 R)\}}{(dA/dD)\{(\alpha V - V_w)(V - V_w) + (1 - \alpha)V V_w\} - gA \cos \theta} \dots \dots (3)$$

and

$$\frac{dD}{d\xi} = - \frac{gA\{\sin \theta - (V^2/C^2 R)\}}{(dA/dD)\{(\alpha V - V_w)(V - V_w) + (1 - \alpha)V V_w\} - gA \cos \theta} \dots \dots (4)$$

In reality, the wave velocity may not be considered constant in a channel of irregular section, but the treatment of analysis will be proceeded from a macroscopic aspect for mean flows. Eliminating V from Eqs. 3 and 4, the steadied wave pattern observed from the moving origin can be expressed by

$$\frac{dD}{d\xi} = - \frac{gA(\sin \theta - \{(V_w A - K^2/C^2 R A^2)\})}{(dA/dD)\{(\alpha K^2/A^2) + (1 - \alpha)V_w^2\} - gA \cos \theta} = - \frac{f_1(D)}{f_2(D)} \dots \dots (5)$$

in which K is a progressive discharge rate defined by $(V_w - V)A = \text{a constant}$. As seen in Eq. 5, the wave pattern of a roll wave cannot be derived in a periodic form observed in laboratories, and thus the solution for roll waves must be obtained as a discontinuous wave pattern by combining each of water surface profiles through the shock condition as Dressler did. Dressler also obtained a continuous periodic solution of roll waves being in forms of the elliptic function as the second order solution of the shallow water wave theory. The actual wave pattern, however, observed in channels is rather in a similar form to discontinuous periodic solutions.

The basic concept of Thomas obtained by his specified experimental works indicates that the flow involves a control section in each water surface profile, as the flow regime changes from tranquil to shooting, so that in Eq. 5, both numerator and denominator become simultaneously zero and all the hydraulic characteristics of roll waves can be uniquely determined at this section. The resulting expressions for the mean velocity and the hydraulic radius or the water depth in specified cases at the control section are

$$\frac{V_o}{V_w} = \frac{\alpha(dA/dD)_o - \sqrt{\alpha(\alpha - 1)(dA/dD)_o^2 + \{(dA/dD)_o S_o/N_F^2\}}}{\alpha(dA/dD)_o - (S_o/N_F^2)} \dots \dots (6)$$

and

$$\frac{R_o g \cos \theta}{V_w^2} = \frac{1}{N_F^2} \left[\frac{\alpha(dA/dD)_o - \sqrt{\alpha(\alpha - 1)(dA/dD)_o^2 + \{(dA/dD)_o S_o/N_F^2\}}}{\alpha(dA/dD)_o - (S_o/N_F^2)} \right]^2 \dots (7)$$

in which S is the wetted perimeter, N_F denotes the Froude number defined by $V_o/(g R_o \cos \theta)^{1/2}$, and the subscript o indicates the values at the control section which are eventually equal to the quantities in uniform flow. The progressive discharge rate can also be calculated by the use of Eqs. 6 and 7.

If the flow is laminar in a rectangular channel, the normal mean velocity of flow, the normal depth of water, and the progressive discharge rate are

explicitely indicated as functions of the wave velocity and Froude number as follows:

$$\frac{V_o}{V_w} = \frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)} \dots\dots\dots (8)$$

$$\frac{D_o g \cos \theta}{V_w^2} = \frac{1}{N_F^2} \left\{ \frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)} \right\}^2 \dots\dots\dots (9)$$

and

$$\frac{K g \cos \theta}{V_w^3} = \frac{1}{N_F^2} \left\{ \frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)} \right\}^2$$

$$1 - \left\{ \frac{(6/5) - \sqrt{(6/25) + (1/N_F^2)}}{(6/5) - (1/N_F^2)} \right\} \dots\dots\dots (10)$$

Water-surface profiles as each wave pattern of roll waves are obtained by integrating Eq. 5 upstream and downstream from the control section. Although water-surface-profile equations cannot be integrated analytically in general cases, those for laminar flows and for turbulent flows, characterized by a constant Chézy coefficient in rectangular channels, are integrable. In cases of two-dimensional laminar flows, the resulting water-surface profile is indicated in an inverse functional form as follows:

$$\zeta = \frac{1}{\sin \theta} \frac{D_A^2 \cos \theta + (6/5)(K^2 D_A / g D_o^2) + (6/5)(K^2 / g D_o)}{D_A - D_B} \log_e \frac{D - D_A}{D_o - D_A}$$

$$- \frac{D_B^2 \cos \theta + (6/5)(K^2 D_B / g D_o^2) + (6/5)(K^2 / g D_o)}{D_A - D_B} \log_e \frac{D - D_B}{D_o - D_B}$$

$$+ \cos \theta (D - D_o) \left\{ \dots\dots\dots (11) \right.$$

in which the integral constant is selected so that $D = D_o$ at $\zeta = 0$, and D_A and D_B are

$$\frac{D_A}{D_o} = \sqrt{\frac{9}{20} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}}} - \frac{1}{2} \dots\dots\dots (12)$$

$$\frac{D_B}{D_o} = - \frac{9}{20} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}} + \frac{1}{2} \dots\dots\dots (13)$$

and $D_o > D_A > 0 > D_B$.

The complete discontinuous solution of roll waves can be obtained by combining each of water-surface profiles through the shock condition, as will be described in the following.

The shock condition for open-channel flows involving a moving discontinuity is expressed by the following three conditions:

Mass conservation:

$$-\rho A_b(V_w - V_b) = -\rho A_f(V_w - V_f) = M \dots \dots \dots (14)$$

Momentum conservation:

$$M(V_w - V_f) - (V_w - V_b) = \rho g \cos \theta \{A_f(D_f - D_{Gf}) - A_b(D_b - D_{Gb})\} \dots \dots (15)$$

Energy dissipation rate:

$$P = \frac{dE}{dt} = \frac{M g \cos \theta}{2 A_f A_b} \{A_b^2(D_b - D_{Gb}) - A_b A_f(D_b + D_{Gb}) + A_b A_f(D_f + D_{Gf}) - A_f^2(D_f - D_{Gf})\} \dots \dots \dots (16)$$

in which D_G is the distance from the channel bed to the centroid of flow area, and the subscripts of f and b indicate the values at the lowest and highest points of water-surface profiles. Using the expression for progressive discharge rate and Eqs. 14 and 15, the relationship between the lowest and highest depths of a wave becomes

$$K^2 \{ (1/A_b) - (1/A_f) \} = g \cos \theta \{ A_f(D_f - D_{Gf}) - A_b(D_b - D_{Gb}) \} \dots \dots (17)$$

Denoting the wave length of a roll wave by L , the following relationship must exist between two successive waves:

$$\zeta(D_{n+1}) = L + \zeta(D_n) \dots \dots \dots (18)$$

and at the shock front

$$\zeta_{D_n}(D_b) = \zeta_{D_{n+1}}(D_f) \dots \dots \dots (19)$$

If the water-surface profile for a particular roll wave is obtained by integrating numerically Eq. 5, the complete solution of discontinuous periodic pattern of roll waves will also be calculated graphically or numerically. In cases of two-dimensional laminar flows, the wave length of a roll wave is explicitly determined by

$$L = \frac{1}{\sin \theta} \left\{ \frac{D_A^2 \cos \theta + (6/5)(K^2 D_A / g D_0^2) + (6/5)(K^2 / g D_0)}{D_A - D_B} \log_e \frac{D_b - D_A}{D_f - D_A} - \frac{D_B^2 \cos \theta + (6/5)(K^2 D_B / g D_0^2) + (6/5)(K^2 / g D_0)}{D_A - D_B} \log_e \frac{D_b - D_B}{D_f - D_B} + \cos \theta (D_b - D_f) \right\} \dots \dots \dots (20)$$

The criterion for roll waves to maintain their final wave pattern, as described in the foregoing, will be studied next. It is of common observation that the wave profile of a roll wave is monoclinal increasing from the rear side to the front side, so that the following relationship for the water depth must be obtained at the control section, when the waves are observed from the moving origin.

$$\lim_{D=D_0} \frac{dD}{d\xi} \geq 0 \quad (21)$$

With the aid of Eq. 5, the above condition for laminar flows characterized by the parabolic velocity profile and $\alpha = 1.2$ becomes

$$\frac{2 m V_0}{V_w - V_0} \geq 1 \quad (22)$$

or, expressing in terms of Froude number,

$$N_F^2 \geq S_0 / \left(\frac{dA}{dD} \right)_0 (4 m^2 - 0.8 m - 0.2) \quad (23)$$

in which m is the shape parameter defined by $1 - R_0(ds/dA)_0$ and it gradually increases to unity with increase in width of a channel.

If the flow is turbulent, the criterion is also calculated in the same manner, and it is

$$\frac{m V_0}{2 V_w} \left\{ \frac{2 R_0}{C_0} \left(\frac{dC}{dR} \right)_0 + \frac{2}{m} + 1 \right\} \geq 1 \quad (24)$$

for the channels in which the flow area is proportional to a power of the water depth. To obtain the criterion in a more familiar fashion for hydraulic engineers, the velocity formula of Vedernikov's type will be introduced as

$$V_0^a = \beta R_0^{1+b} \sin \theta \quad (25)$$

in which a , b and β are constants determined by given flow and channel characteristics, so that the Chézy's coefficient C becomes

$$C_0 = \beta^{1/a} R_0^{2(1+b)-a/2a} (\sin \theta)^{2-a/2a} \quad (26)$$

Inserting Eq. 26 into Eq. 24, the resulting condition becomes

$$V_e = \frac{(1+b)mV_0}{a(V_w - V_0)} \geq 1 \quad (27)$$

or, expressing in terms of Froude number,

$$N_F^2 \geq S_0 / \left(\frac{dA}{dD} \right)_0 \left\{ m^2 \left(\frac{1+b}{a} \right)^2 - 2(\alpha-1)m \left(\frac{1+b}{a} \right) - (\alpha-1) \right\} \quad (28)$$

Eq. 27 is evidently the Vedernikov number for the initial instability of shooting flows. The last of the writers⁹ also derived the same criterion by deduc-

ing the initial condition for continuous time growth of an infinitesimally small disturbance with the use of Eqs. 1 and 2. A. Craya¹² and Dressler¹³ studied the similar criteria, and in specified cases of particular velocity formulas Jeffreys¹⁴ and Keulegan and Patterson¹⁵ obtained the instability conditions. Of most significance, in deduction of the foregoing maintenance conditions for roll waves being in their final wave pattern, is that two conditions for maintenance of roll waves and for initial instability of shooting flows are exactly the same expression of Vedernikov, regardless of essentially different approaches in deduction. The formation of roll waves, therefore, will result from the instability of open-channel flows as many hydraulic engineers described.

The previous explanation for the hydraulic characteristics of roll waves is a summary of our past analytical treatment developed by means of the basic concepts of Thomas and Dressler. The experimental works for the characteristics of roll waves, to clarify the hydraulic behaviors of waves in definite forms and to verify the theoretical deduction of roll wave characteristics, were also paralleled by the writers in the hydraulics laboratory of Kyoto University. Two test flumes of a rectangular section were used for this purpose, and in the earlier stage of research progress, which was mainly subject to laminar roll waves, the aluminum channel of 20 ft in length was used, whereas the experimental runs for turbulent roll waves, which are called as slug flows by the author, were made in the lucite flume of 36 ft in length and of variable slopes from horizontal to 30°. Continuous recordings of wave profiles of roll waves and of wave velocities were made by the electronic wave recorders of resistance type, and the discharge measurement was made by volume metering of water at the downstream end. The details of experimental procedures were reported in the writers' paper.⁴ Comparing the theoretical results described in the foregoing, the experimental behaviors of roll waves obtained by the writers will be presented in the following. Data of the author and Binnie¹⁶ will be also supplemented.

The criterion for initial instability and formation of roll waves will be treated first. For the sake of simplicity in illustration, the flow is assumed two-dimensional. In this case, Eq. 23, which indicates the criterion for laminar flows, becomes

$$N_F = 0.577 \quad \text{or} \quad \frac{1}{\tan \theta R_D} = 1 \dots \dots \dots (29)$$

in which R_D is the Reynolds number given by $V_0 D_0 / \nu$, and the same criterion for turbulent flows is also in terms of Froude number,

$$N_F^2 = 1 / \left\{ \left(\frac{1+b}{a} \right)^2 - 2(\alpha-1) \left(\frac{1+b}{a} \right) - (\alpha-1) \right\} \dots \dots \dots (30)$$

¹² "The Criterion for the Possibility of Roll Wave Formation," by A. Craya. Proc. of the Gravity Wave Symposium, NBS, 1951.

¹³ "Resistance Effects on Hydraulic Instability," by R.F. Dressler and F.V. Pohle. Comm. on Pure and Applied Math., Vol. 6, No. 1, 1953.

¹⁴ "The Flow of Water in an Inclined Channel of Rectangular Section," by H. Jeffreys. Phil. Mag., Series 6, Vol. 49, 1925.

¹⁵ "A Criterion for Instability of Flow in Steep Channels," by G. H. Keulegan and G. W. Patterson. Trans. AGU, Part II, 1940.

¹⁶ "Instability in a Slightly Inclined Water Channel," by A.M. Binnie. Journal of Fluid Mech., Vol. 5, Part 4, May, 1959.

and consequently, it depends on the velocity formulas like Chézy's or Manning's and the momentum correction factor. Since the Froude number is expressed as a function of the Reynolds number, Manning's roughness coefficient, slope and kinematic viscosity, when the Manning formula is used, the instability criterion can be graphically illustrated in terms of the slope and Reynolds number as parametric expressions of the roughness and the momentum correction factor. Fig. 1 illustrates the criterion for instability of flows and formation of roll waves obtained in the preceding manner under the assumption that the kinematic viscosity is 0.01 sq cm per sec. Two zones of stable and unstable flows will be apparently observed in the figure. In open-channel flow, the transition from laminar to turbulent flows takes place for Reynolds number from 500 to 1,500, as often indicated by many engineers.

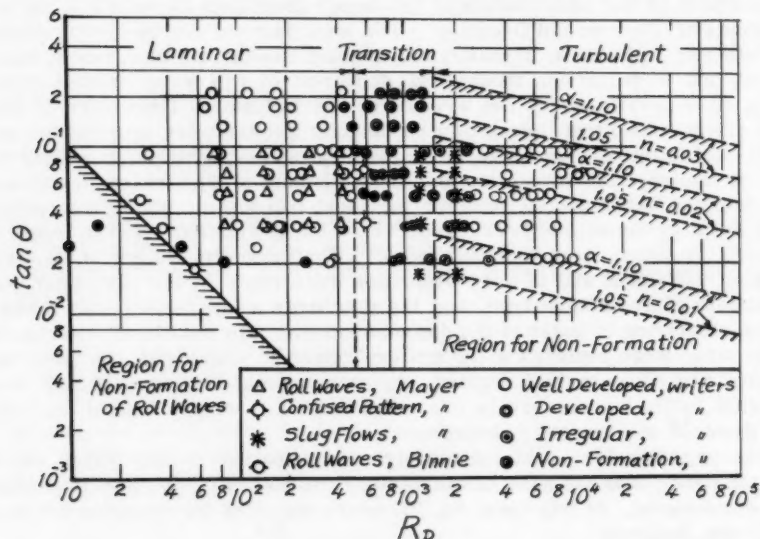


FIG. 1.—REPRESENTATION OF CRITERION FOR FORMATION OF ROLL WAVES.

The second of the writers⁷ found that the Manning formula was not suitable in the transition region, but $(1 + b)/a$ was commonly very small and it became negative in particular cases depending on the channel slope, through the experimental research. The formation of roll waves, therefore, will not result.

Now, let us consider the behavior of roll waves in formation when the flow is carried in an open channel. For clarification in explanation, the channel slope is assumed 0.02. When the regulating valve for discharge is opened from closure, the flow begins to be carried. The roll waves are not observed until the Reynolds number becomes 50, as previously indicated. With increase in discharge, the laminar roll waves become appreciable. Once the Reynolds number increases to 500, so the transition from laminar to turbulent flows begins to take place and the roll waves disappear. This phenomenon may be corresponding to that called the confused pattern by the author. With the further increase in discharge, the flow becomes completely turbulent, and

again the roll waves are predominant in forms of bore if the channel roughness is approximately less than 0.01 in Manning's roughness coefficient, whereas for rough channels in which the coefficient is larger than 0.01, the roll waves can not be observed. When the discharge is still increased, the roll waves ultimately appear even in such rough channels. This fact is evidently equal to the results obtained by Thomas,¹⁰ Dressler¹¹ and others.

In Fig. 1, our experimental data for roll waves are plotted, adding supplemental data of the author and Binnie. It is then concluded that the foregoing treatment for the formation of roll waves is verified by the experimental observations. Evidently, the law of resistance and the momentum correction factor play a very important role in the criterion for formation of roll waves. Fig. 2 indicates the criterion in terms of velocity formula and momentum

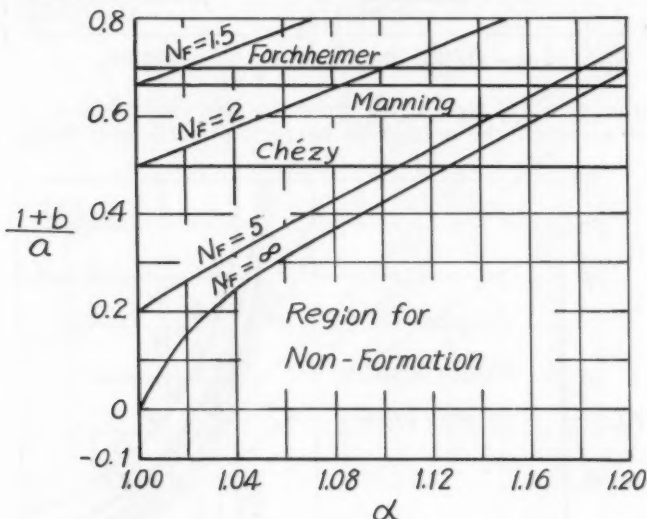


FIG. 2.—ILLUSTRATION OF CRITERION FOR FORMATION OF ROLL WAVES IN TERMS OF LAW OF RESISTANCE AND MOMENTUM CORRECTION FACTOR.

correction factor, and it also illustrates that roll waves cannot be observed in the transition region from laminar to turbulent flows, as $(1+b)/a$ is very small in this region.

In the same manner, the criterion for roll wave formation in triangular channels will be obtained as follows.

$$N_F^2 = 4/\sin \phi \left\{ \left(\frac{1+b}{a} \right)^2 - 4(\alpha-1) \left(\frac{1+b}{a} \right) - 4(\alpha-1) \right\} \dots \dots (31)$$

in which ϕ is a half of the angle between two side walls. Fig. 3 indicates the same criterion for triangular channels, and it is obvious that the roll-wave formation in triangular channels cannot be compared with that in rectangular channels.

The behavior of wave velocity of roll waves when the wave pattern becomes ultimately stable will be described next. By the use of the expressions for

progressive discharge rate and for the mean velocity of flow at the control section, the wave velocity is determined. Introducing the dimensionless wave velocity by $V_w' = V_w / g q_0 \cos \theta)^{1/3}$ for rectangular channels, in which q_0 is the discharge per unit width for uniform flow, V_w' is

$$V_w' = \{ \alpha - 1 + \sqrt{\alpha(\alpha - 1) + (1/N_F^2)} \}^{1/3} K^{1/3} \dots (32)$$

As the dimensionless progressive discharge rate K' , in which

$$K' = K \frac{g \cos \theta}{V_w^3}$$

$$= \frac{1}{N_F^2} \left\{ \frac{\alpha - \sqrt{\alpha(\alpha - 1) + (1/N_F^2)}}{\alpha - (1/N_F^2)} \right\}^2 \left\{ 1 - \frac{\alpha - \sqrt{\alpha(\alpha - 1) + (1/N_F^2)}}{\alpha - (1/N_F^2)} \right\} \dots (33)$$

is a function of α and N_F , so the dimensionless wave velocity is also expressed in terms of the momentum correction factor and Froude number.

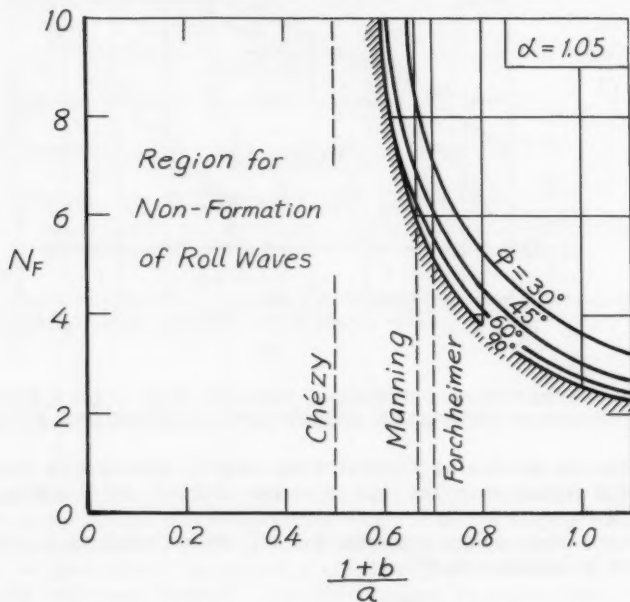


FIG. 3.—CRITERION FOR FORMATION OF ROLL WAVES IN TRIANGULAR CHANNELS.

Fig. 4 shows the behaviors of V_w' for both laminar and turbulent flows. In the same figure, our experimental data are plotted with data of the author and Binnie, and it is found that the experimental verification, to the theoretical approach, can be made.

By the similar procedures, the behaviors of wave length, wave period, and wave height as the hydraulic characteristics are determined. Introducing the

following dimensionless parameters of wave length, wave period and wave height,

$$L' = L \tan \theta (g \cos \theta / V_w^2), \dots \dots \dots (34a)$$

$$T' = L' / V_w', \dots \dots \dots (34b)$$

and

$$D_b' = D_b (g \cos \theta / V_w^2), \dots \dots \dots (34c)$$

Figs. 5-7 are graphically obtained by the numerical calculation, using the foregoing relationships. The actual wave height at the frontal side is always

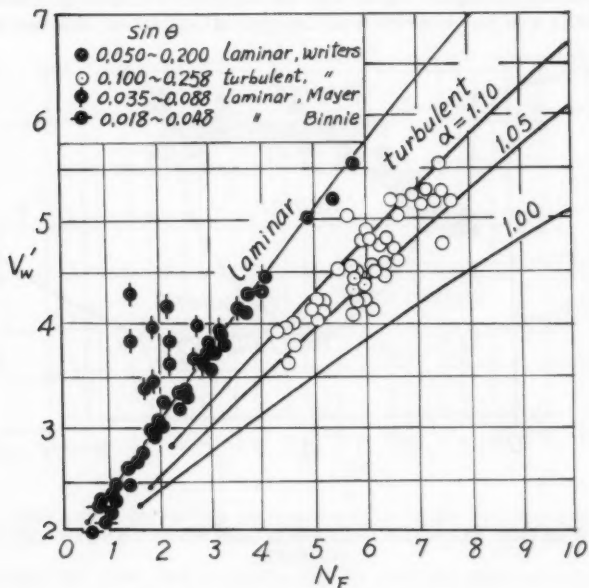


FIG. 4.—BEHAVIORS OF DIMENSIONLESS WAVE VELOCITY.

less than the calculated one, because the calculated wave profiles have sharp wave crests due to the discontinuous shock front, whereas the actual wave crest is round probably because of the capillary effect and the existence of vorticities at the frontal side.

Other hydraulic characteristics in flow, due to the formation of roll waves, are calculated in the same way. However, these are not concerned herein, and the details of hydraulic characteristics in flow will be seen in the writers' paper.⁴⁻⁷

Based on the results derived and the knowledge obtained by the writers through the research works, the following 16 items are presented:

1. The author states that the roll wave is the result of interaction between the surface tension and the gravity force in a slightly disturbed flow. How-

ever, the hydraulic characteristics of roll waves in laminar flow can be theoretically analyzed as presented in the foregoing, without introducing the effect of surface tensions, and the results derived theoretically are satisfactorily in agreement with the experimental ones obtained by the writers and Binnie. From this fact it may be concluded that the effect of capillary forces on the formation and the characteristics of roll waves is quite little or rather negligible practically, except in cases of extremely steep slopes and of very low Reynolds number.

2. The author concludes that slug flows result from instability which causes the transition from supercritical laminar to turbulent flow. Slug flows, defined by the author, seem to be corresponding to incomplete roll waves at low Reynolds number in turbulent flow just changed from laminar to completely turbulent through transition in its flow regime. The fact that roll waves and slug flows are not observed at the transition from laminar to turbulent

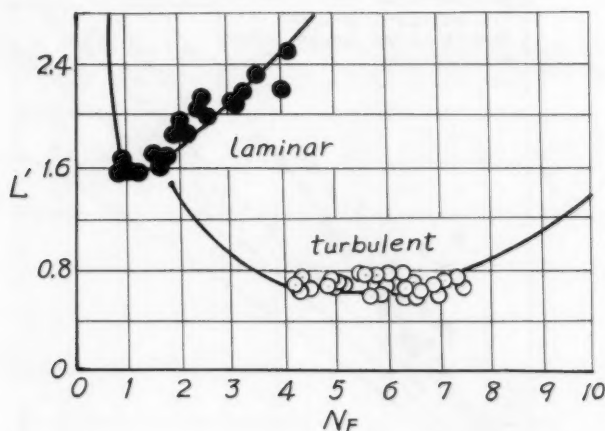


FIG. 5.—RELATION BETWEEN DIMENSIONLESS WAVE LENGTH AND FROUDE NUMBER.

flow is caused by the different nature of flow and thus the different laws of resistance as described in the foregoing. Slug flows, therefore, should be explained as kinds of roll waves which result from instabilities of flow itself, at low Reynolds number in turbulent flow as well as roll waves in laminar flow.

3. The author's Eq. 11 is questionable, whether it represents the criterion for formation of roll waves and slug flows or not. According to Eq. 11, roll waves are formed in laminar flow, when $N_F < 2$. However, the author's Fig. 12 indicates that roll waves were observed even when $N_F > 2$, and the writers also obtained the same results as shown in the figures illustrated in the foregoing. Consequently, $N_F = 2$ is not a criterion for formation of roll waves in laminar flow.

4. The author applied the criterion for the instability of turbulent flow and the formation of turbulent roll waves,

$$S = 4 \text{ g/C}^2 \dots\dots\dots (35)$$

which was derived by the use of the Chézy formula, to laminar flow. Since, however, Eq. 35 was derived by putting $C = \text{constant}$, the author's Eq. 21 should not be used to obtain Eq. 17a from Eq. 35.

5. In regard to the effect of surface tensions, the author suggests that the velocity of roll waves is expressed by

$$V_w = V + C_{\min}. \quad (36)$$

This relation is based on the idea that the wave length decreases and conversely the wave height increases with time, and ultimately the condition of minimum celerity C_{\min} is attained. However, it is evident from Figs. 20 and 23 that the wave velocity, the wave length, and the wave height actually increase until the terminal values are reached with the lapse of time. This fact is inconsistent with the author's suggestion.

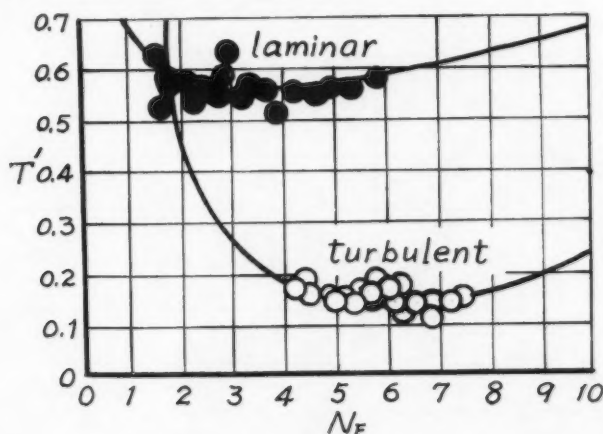


FIG. 6.—RELATION BETWEEN DIMENSIONLESS WAVE PERIOD AND FROUDE NUMBER.

6. For laminar flow, the author's Eq. 19 verifies that the plot of $N_F/S^{1/2}$ against R_D shown in Fig. 14 is correct as illustrated again in Fig. 18. In the same manner, since Eq. 19 is written as

$$N_F/\sqrt{S} = \text{const.} (g, \nu, \sigma/\rho) \cdot (N_w/S^{1/3})^{3/10} \quad (37)$$

The author's Eq. 15 is suggested to plot $N_F/S^{1/2}$ against $N_w/S^{1/3}$.

7. Graphical expressions for N_F against R_D in Fig. 12 and for N_F against N_w in Fig. 13 are the indication of the laws of resistance in each regime of flow, and the hydraulic characteristics of roll waves and slug flows are not shown. Therefore, the author's conclusion is not derived from these figures, that the surface tensions and viscous effects predominate in the phenomenon of roll wave formation, and at higher Reynolds numbers, corresponding to the conditions which led to slug flows, viscous and surface tension effects decreased and, finally, became negligible.

According to the results of analysis by the writers, the viscous effect on the characteristics of roll waves in laminar flow is introduced indirectly as

Froude number from the relation of Eq. 19 because all the dimensionless quantities of roll wave characteristics are functions of the Froude number, as shown previously. On the other hand, characteristics of roll waves (slug flows) in turbulent flow are not influenced by the viscosity as far as Manning's formula is used and its roughness coefficient is assumed independent on the viscosity. Actually, however, the viscous effect may exist more or less if the channel bed condition is a hydraulically smooth boundary.

8. If N_D and F_{Cr} are put in a form of

$$N_D = \gamma F_{Cr}^{\delta} \dots \dots \dots (38)$$

it is found after simple computation using the author's Eqs. 19 and 37 that the value of δ should be $1/4$ for laminar flow instead of 0.218 in the author's Eq. 32.

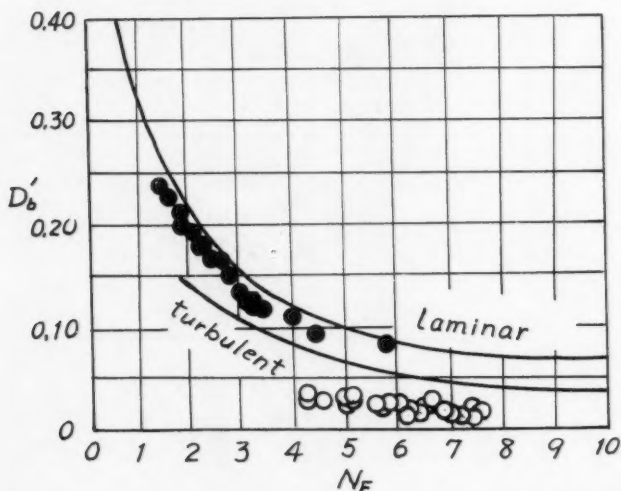


FIG. 7.—RELATION BETWEEN DIMENSIONLESS WAVE HEIGHT AND FROUDE NUMBER.

9. In obtaining the minimum slope $0.029 \sim 3\%$ for the formation of roll waves, the criterion to laminar flow

$$S \geq \frac{12}{R_D} \dots \dots \dots (39)$$

was applied. This criterion is corresponding to

$$N_F \geq 2 \dots \dots \dots (40)$$

and based on

$$V + \sqrt{gD} \leq V_s \dots \dots \dots (41)$$

However, Fig. 17 indicates that roll waves develop even when $N_F < 2$ and it is evident that the latter is inconsistent with the author's idea for the roll wave formation.

10. In deriving the minimum slope 0.01 for the formation of slug flows, Eq. 39 was also applied. At the Reynolds number 1,200, which the author used as a lower limit of Reynolds numbers, the flow regime will be the transition from laminar to turbulent flow. Therefore, it is doubtful that Eq. 39, which was derived by using the law of resistance to laminar flow, can be applied to the formation of slug flows.

11. The writers are much interested in Fig. 19 which shows the location of initiation of roll waves and slug flows. This figure indicates the fact that roll waves and slug flows cannot be formed in the transition from laminar to turbulent flow, otherwise very long distances are needed for their development. This fact was already pointed out by the writers in their Fig. 1.

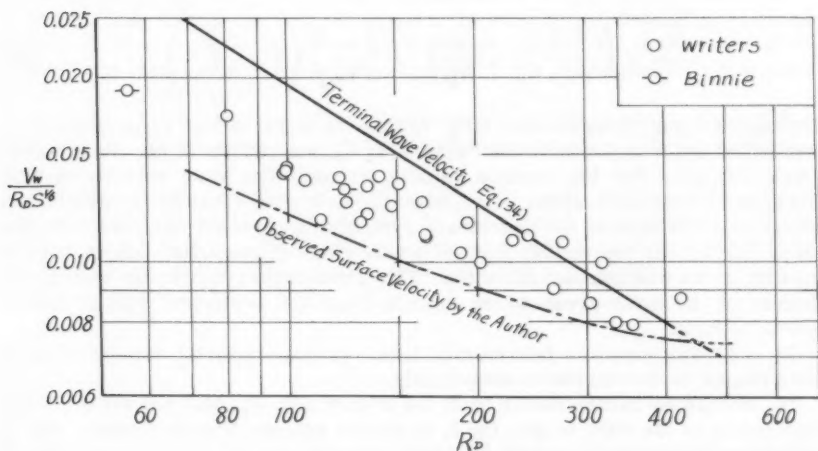


FIG. 8.—PLOTS OF THE WRITERS' AND BINNIE'S DATA OF ROLL WAVE VELOCITY IN FORM PROPOSED BY THE AUTHOR.

12. The author suggests that the wave velocity of roll waves V_w is equal to $V + C_{min}$, in which $C_{min} = 0.763$ ft per sec. Although the value of $V_w - V$ obtained from the experimental data is of the same order in magnitude as C_{min} , his suggestion to the wave velocity is too intuitive and the effect of capillary is given too much evaluation.

13. The writer's and Binnie's data of the wave velocity in laminar flow are presented in Fig. 8 in the same way as plotted in Fig. 24. It is found from the figure that the author's data are larger than the writer's and Binnie's, except at Reynolds numbers of 280 to 420. This disagreement in addition to that in Fig. 4 cannot be understood by the writers.

14. The ratio of the wave velocity V_w to the surface velocity V_s in laminar flow is written, from Eq. 8, as

$$\begin{aligned}
 \frac{V_w}{V_s} &= \left(\frac{V}{V_s} \right) \left(\frac{V_w}{V} \right) \\
 &= \frac{2}{3} \frac{6}{5} + \sqrt{\frac{6}{25} + \frac{1}{N_F^2}} \\
 &= \frac{4}{5} + \frac{2}{3} \sqrt{\frac{6}{25} + \frac{3}{S R_D}} \dots\dots\dots (42)
 \end{aligned}$$

This relation shows that the ratio V_w/V_s is a function not only of R_D but also of the slope S . Therefore, the slope should be introduced as a parameter in Fig. 25. It is not difficult to find from Eq. 42 that the value of V_w/V is always greater than unity and increases with decrease in R_D if the slope is constant. This trend agrees with that shown in Fig. 25.

15. In turbulent flow, the ratio of the wave velocity (slug flows) to the surface velocity is expressed, from Eq. 6 if assuming $\alpha = 1$, as

$$\frac{V_w}{V_s} = \left(\frac{V}{V_s} \right) \left(1 + \frac{1}{N_F} \right) = \left(\frac{V}{V_s} \right) \left(1 + \frac{3}{S R_D} \right) \dots\dots\dots (43)$$

Although it seems possible from Eq. 43 that the value of V_w/V_s becomes less than unity, which depends on the value of V/V_s , the results of his observation, which indicates that the surface velocity exceeds the wave velocity of slug flows, can not be understood. If so, when the flow with waves is transferred to steady-state by moving the coordinate system at a constant wave velocity, the water surface in the virtual flow of water running upstream has to have a velocity in the downstream direction. This phenomenon is possible only in the vicinity of the wave crest which results from the horizontal vortex due to hydraulic jump.

By only this possible behavior of flow, the experimental results of slug flows cannot be interpreted analytically.

16. It may be rather better than the plot in Fig. 26, that the ratio of the wave height to the wave length D_h/L is plotted against Froude number with a parameter of the slope because both $D_h g \cos \theta / V_w^2$ and $L \tan \theta g \cos \theta / V_w^2$ are functions of N_F as presented previously.

SPILLWAY DESIGN FOR PACIFIC NORTHWEST PROJECTS^a

Discussion by B. Michel and A.R. Gagnon

B. MICHEL¹ and A.R. GAGNON,² A.M. ASCE.—This paper has brought up one of the oldest, but still one of the most discussed, problems of hydraulic engineering: the design of spillway crests.

The author should be commended for having presented his interesting views on this problem and for the attempt to standardize the design of spillways.

In this last regard the writers agree fully with Mr. Webster that semicircular pier noses induce small flow contractions and that a 1 on 12 taper on the downstream end of gate slots is generally one of the best means to lower cavitation risks at the slots.

However, the writers believe that a second thought should be given before generalizing the process of designing spillway crest for 75% of the nominal head.

An outstanding contribution in that field has been made recently by R. Lemoine³ and the writers believe that he has given a very general basis for spillway crest design. Mr. Lemoine proposes to adopt a cavitation factor for spillway crests in the same manner as for the design of turbines:

This factor is

$$\sigma = \frac{p - p_v}{w \frac{v^2}{2g}} \dots \dots \dots (1)$$

in which p_v is the water vapor pressure at the site, p represents the absolute minimum pressure on the spillway crest, $v^2/2g$ is the velocity head at the point of minimum pressure, and w denotes the specific weight of water.

This factor takes into account, not only the actual permanent negative pressure in the dangerous area, but also the danger of having the velocity head transformed into an additional negative pressure in low pressure nuclei eddies, formed behind asperities of the spillway surface. Thus, the factor should cover all conditions of cavitation on spillway profiles.

The design of any spillway can then be made with the following considerations:

$\sigma = 0$: certain cavitation;

^a August, 1959, by Marvin J. Webster.

¹ Prof. of Hydr. Engrg., Laval Univ.

² Lecturer in Fluid Mech., Laval Univ., Quebec City.

³ "Cavitation on Weir Crests," by R. Lemoine. Proceedings from 6th Meeting of the IAHR, The Hague, 1955.

- $\sigma = 1$: cavitation probably impossible, even if all the velocity head was transformed into additional negative pressure; and
 $\sigma = 0.5$: acceptable risk of cavitation if this working condition occurs rarely and if the spillway surface is very smooth.

For a standard ogee spillway it is easy to know the minimum pressure at the crest,^{4,5,6} the velocity head, and to design the spillway with the help of the cavitation factor. It is indeed interesting to note that the foregoing references show that, for an overdesign of 75%, the cavitation factor, after a "critical design head," is invariably zero while the minimum pressure becomes equal to the vapor pressure. It might be also noticed that the classical form is not generally the one that will give the highest discharge coefficient for a certain value of the cavitation factor. Model tests must be made in each case.

The model results given in Mr. Webster's paper, for the Chief Joseph Spillway, show a minimum pressure of -23 ft of water with no piers and -9 ft with piers. These results are somewhat higher than those obtained on similar spillway profiles by many investigators. This could probably be explained by the fact that very small differences in the geometry of each project have an important effect on pressure values, and also by the very high difficulty involved in measuring the peak of rapidly varied pressure distribution on a scale model. The writers do not believe that the results for this spillway are representative of all those of the same type.

Taking the last value for the minimum pressure of -9 ft with an operating head of 54 ft it is found that the cavitation factor would then be 0.38, which is somewhat lower than could be permissible with Mr. Lemoine's theory.

The author was fully aware of some cavitation risks and states that due to the low predicted frequency of the maximum discharge, the cost saved in initial construction more than offsets the cost of infrequent repairs resulting from damage due to negative pressure.

The writers agree with him and this could easily be taken into account by choosing, in each case, an accepted value of the cavitation factor. The writers strongly believe that this would be more logical than to accept a percentage head design irrespective of the value of the head itself, the actual shape and pressures on the crest.

⁴ "Untersuchungen an Überfällen," by O. Dillmann. Mitteilungen des hydraulischen Instituts, Munich #7, 1933.

⁵ "Engineering Hydraulics," Hunter Rouse (Editor), Chapter VIII.

⁶ "Model Research on Spillway Crest," by Rouse and Reid. Civ. Engrg., Vol. 5, No. 1, 1935.

FLOOD CONTROL ASPECTS OF CAUCA VALLEY DEVELOPMENT^a

Discussion by Julio Escobar-Fernandez

JULIO ESCOBAR-FERNANDEZ.¹—Three aspects from the paper by Messrs. Kirpich and Ospina could be considered. First, the significance and importance, in the development of the so-called underdeveloped countries, of the establishment of regional autonomous corporations of development; second, the technical difficulties that have to be faced by engineers interested in water problems in such countries, where hydrological data is invariably scarce, or of doubtful value and inconsistent; and third, the technical features of a big hydroelectric project, fleetingly mentioned in the paper, which could be developed by diverting the waters of the Cauca River to the Pacific coast, a system that could be classified as one of the important energetic possibilities, not only of Latin America but of the world, with the particular advantage of its proximity to the sea, a feature that makes it especially attractive for the establishment of heavy electrochemical industries.

Although the three aspects might be of general interest, due to limitations of space, only the first one will be discussed, at present. Some of the general principles governing the policies and the organization of regional corporations devoted to local development are set forth, and some emphasis is placed on the importance of such bodies as helpful instruments in the execution of wider development programs, usually national in scope. The timeliness of the subject seems especially relevant today (1960), when better management for economic improvement constitutes a daily concern for the governments in the less developed countries.

Colombia is a country with rich natural resources that can be developed to create an economically solid nation, thereby raising the standard of living of its people. To reach these two objectives, in time and without waste of effort, it is considered that a plan for the development and effective use of those resources is indispensable, as is the implementation of the plan. In Colombia, the Central Government has already launched such a program and the recently organized Planning Unit, directly responsible to the President of the Republic, is a fundamental step towards the coordinated execution of the plans that have been already formulated or which are under study. As a starting point, a better orientation of government investments is considered one of the primary policies of development, because such investments as in transportation, energy production, irrigation and public utilities are or should be the foundation on which a solid economy must rest.

On the other hand, the expansion of public enterprise, which constitutes the other fundamental basis on which the progress of nations depends, must be

^a September, 1959, by Phillip Z. Kirpich and Carlos S. Ospina.

¹ Escobar, Venegas y Rodriguez, Ingenieros Ltda., Bogota, Colombia.

stimulated not only by the facilities provided by those public investments, but by the creation, through government measures, of a minimum of general conditions favoring private investment.

As one of its immediate tasks, the new Planning Unit will therefore have to establish a balance between all resources available—economical, physical and human—and the many needs of the country. From this overall picture, a more or less rigid schedule of priorities must be determined, in order to establish how and where such resources can be spent most usefully and effectively.

Once a general plan for the whole country has been defined, the next step should be to project it on a sectorial basis, to use the economist's jargon, or on a regional basis, if the characteristics of the country so allow, as is the case in Colombia.

A program of integrated regional development, such as the one undertaken in Colombia in the valley of the Cauca River has, among others, the following advantages:

1. It permits the establishment of a kind of experimental field where several techniques of development can be tested. In this way, the selected zone becomes a demonstration area, where direct experience is the best school;
2. financial problems may be of a reduced magnitude in comparison with a nation-wide program of development.
3. administrative problems are also reduced; and,
4. in developing a particular area, the economic benefits spread throughout the country.

When in 1954, David Lilienthal, mentioned by the authors of the paper under discussion, recommended to the Government of Colombia the creation of an autonomous regional authority,² as a first stage in its wider program of development, he based his recommendations on several general principles that should guide this type of body, principles that can be resumed as follows:

The initiative for a general program must originate in the region itself. It is therefore indispensable that people in the area be willing to cooperate in the plans, and that enthusiasm for innovation and improvement be general.

Second, the approval and cooperation of the Central Government is also necessary. The Government should be disposed to frame the program with some autonomy, through the creation of an agency or public corporation devoted to that purpose. This agency should be provided with sufficient administrative authority, and with well defined functions and aims. To better define the status of that corporation, it may be useful to quote the words of President Roosevelt when he asked Congress to create the TVA in 1933 (quoted by Mr. Lilienthal, : The authority should be, he said "a corporation clothed with the power of government but possessed of the flexibility and initiative of a private enterprise."

Third, the financial assistance of the Government or of any international organization, such as the World Bank.

Fourth, the creation of a local economic development corporation to promote and stimulate private enterprise in its own development programs.

Finally, it is advisable that the selected region for an experiment of this sort, especially if it is the first to be undertaken in the country, should be one

² "Recommendations on the Establishment of Regional Development Authorities by the Republic of Colombia," by David E. Lilienthal, June 25, 1954.

where the benefits obtained through the techniques of planning, are more or less immediate, in order to overcome the natural tendency of people to be skeptical or even hostile to programs that require a long time to produce definite results.

All of these conditions were satisfied by the Colombian region of the Cauca Valley, and the CVC, the autonomous corporation created in 1955, is beginning to show results in a way that may be considered a real success.

FRICTION FACTORS IN CORRUGATED METAL PIPE^a

Discussion by A.R. Chamberlain and Nicholas Bilonok

A.R. CHAMBERLAIN,¹ A.M. ASCE.—The authors are to be commended for their very careful work in obtaining design information on friction factors for large-diameter corrugated pipes. These data and results presented by the authors will undoubtedly be used by design engineers for several decades, in all cases where pipes of the nature tested are used. This paper represents the result of a tremendous amount of effort on the part of Army Corps personnel.

The writer wishes to extend the data presented by the authors on the basis of clear water, full pipe flow data obtained during 1953-56 in the Hydraulics Laboratory, Colorado State University, Fort Collins, Colo. These data were obtained incidental to certain basic research studies on nominally 12-in. diameter smooth, standard corrugated and helical corrugated pipes.

Each of the types of corrugated metal pipe sections tested were 100-ft long, set on a zero slope. Both corrugated pipes had a clear inside diameter of 12.1 ± 0.03 in., as determined from ten trials on each pipe.

The water was recirculated in a closed system. The discharge was measured by means of a sharp-edged orifice which had been calibrated volumetrically while in place. Discharges were determined, for each discharge-control-valve setting, from the orifice calibration curve using the arithmetic average of four separate readings of the differential head across the orifice.

Hydraulic gradient was determined by means of water piezometers located at 10-ft intervals along the pipeline. The piezometer openings into the pipeline were 3/64-in. in diameter. For each discharge, each piezometer was read four times and the arithmetic average used in hydraulic gradient computations. Upstream end (for 20 pipe diameters) and downstream end (for 10 pipe diameters) piezometers were generally ignored, though recorded, in hydraulic-gradient studies. The piezometers were located on the crest of the corrugations (viewer inside the pipe) after preliminary investigations of the effect of piezometer location along the corrugation, comparable to those of the author's.

The procedure was to first read the orifice manometer, then the piezometer bank and next the orifice manometer again, until each had been read four times during a period of about 1 hr. Next the discharge valve setting was changed, 15 to 20 min. allowed for the flow to stabilize and the next set of data to be obtained.

Fig. C1 shows the two boundaries tested and reported in this discussion. The standard corrugated pipe was zinc coated close-riveted construction.

^a September, 1959, by M. J. Webster and L. R. Metcalf.

¹ Acting Dean, Coll. of Engrg. and Chf., Engrg. Research, Colorado State Univ., Fort Collins, Colo.

The corrugations had a pitch of 0.222 ft along the axis of the pipe. The amplitude of the corrugations from the mean was 0.0208 ft. The crests and troughs were circular arcs with an included angle of approximately 53.73 deg and a radius of 0.0573 ft. Straight tangent sections joined the circular arcs.

The helical corrugated pipe was zinc coated. It had a continuous lock seam joint. The corrugations had a pitch or wave length of 0.167 ft normal to the axis of the corrugation. The amplitude of the helical corrugations was 0.0185 ft, measured from the mean elevation. To complete the corrugations, circular arcs with a radius of 0.0370 ft were joined by straight tangent sections. The

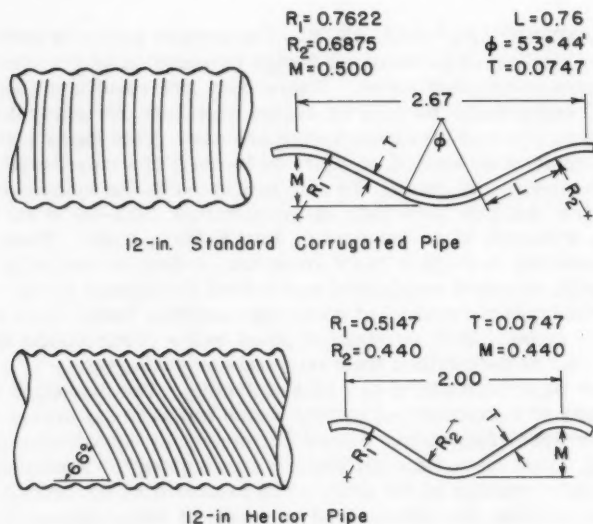


FIG. C1.—DETAILS OF BOUNDARY FORMS.

helix angle, measured between a line drawn along the outer extremity of the pipe parallel to the pipe axis and a line parallel to a corrugation, was 66 deg.

Garde's data were obtained with clear water full pipe flow after the pipe-lines had operated several hundred hours, transporting 0.2 mm mean diameter sand at varying concentrations. Chamberlain's data, designated by a cross, on the Helcor pipe only, were taken just prior to Garde's. The remaining data, all taken by Chamberlain, were recorded with clear water full pipe flows in new pipe.

Derived quantities Re , f , and V were either computed directly or read from graphs prepared for the purpose. In either case, the pipe diameter employed in the analysis was 1.00 ft, slightly less than the inside clear diameter. This magnitude of the diameter was used because (a) it was very convenient, and

more important (b) it was felt that most engineers applying the data would automatically use 1.00 ft in all their design computations.

Tables 1 and 2 summarize the full pipe flow data obtained, admittedly not very extensive, but hopefully of some use to the profession. These data are presented in Fig. C2, for both the standard corrugated and helical corrugated, in a manner comparable to the author's Fig. 10. They are also given in Fig. C3 in the same way as the author's Fig. 11.

Fig. C3 indicates that a recommendation for new 1-ft diameter helical corrugated pipe, of $f = 0.040$, is appropriate. This same figure shows that f is not a function of Reynold's number over the range tested. This may be partially explained by the fact that the corrugations are not normal to the direction of flow in the pipe.

The writer is unable to say why Garde's data, previously unpublished, have such a wide scatter. This is particularly true of the helical corrugated pipe data, since the pipeline was not in use during the interval between Chamberlain's and Garde's used-pipe tests.

Perhaps the most important point to note is that the dashed line portion at the smaller diameter end of the author's Fig. 17 can be extended by plotting in for standard corrugated pipe the tentatively recommended point $f = 0.12$, at $D = 1.0$ ft. Also to be plotted on Fig. 17 for corrugated pipe is the computed (from $f = 0.120$) $n = 0.0255$ at a pipe diameter of 1 ft. The data are insufficient to determine if f is a function of Reynold's number.

Fig. C4 graphically describes the variation of the total differential head between crest and trough piezometers for the standard corrugated pipe tests as a function of Reynold's number. Perhaps more important, however, is the plot, also in Fig. C4, of the ratio of the hydraulic gradients as determined by piezometers at the crest(s) and at the trough(s) of the corrugations, as viewed from inside the pipeline. This plot substantiates that the author's and the writer's method of determining hydraulic gradient from a set of piezometers, located consistently at the same relative location on the corrugations along the pipeline, is sufficiently accurate for engineering purposes over the range of Reynold's number incorporated in the tests.

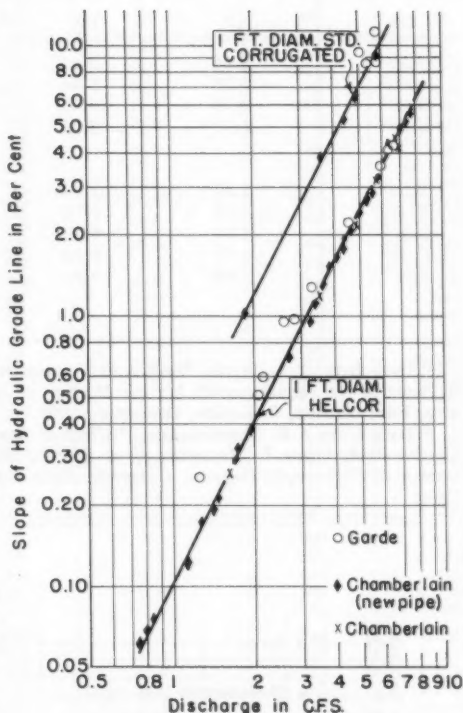


FIG. C2.—DISCHARGE VERSUS HYDRAULIC GRADIENT FOR 1-FT. DIAMETER HELCOR AND CORRUGATED PIPES.

TABLE 1.—SUMMARY OF TEST RESULTS FULL PIPE FLOW
1-FT-DIAMETER CORRUGATED PIPE^a

V, in feet per second (1)	Q, in cubic feet per second (2)	S, in % (3)	Water Temperature in C° (4)	Re x 10 ⁻³ (5)	f (6)
6.37 ^a	5.00	9.24	19.5	590	0.146
6.65 ^a	5.21	8.50	17.7	583	0.124
7.26 ^a	5.70	10.70	19.0	655	0.131
7.26 ^a	5.70	8.50	19.2	660	0.104
				Average	0.126
2.38 ^b	1.87	1.00	27.1	263	0.114
4.45 ^b	3.54	3.80	22.1	430	0.125
5.30 ^b	4.32	5.20	21.6	510	0.121
5.98 ^b	4.78	6.24	23.3	605	0.112
7.23 ^b	5.80	9.00	25.3	770	0.111
				Average	0.117

^a Data from R.J. Garde, "Sediment Transport Through Pipes," Thesis in partial fulfillment of the requirements for the Master of Science Degree, Colorado State University, Fort Collins, Colorado, December 1956.

^b Data from A.R. Chamberlain, "Effect of Boundary Form on Fine Sand Transport in Twelve-Inch Pipes," Dissertation in partial fulfillment of the requirements for the Doctor of Philosophy Degree, Colorado State University, Fort Collins, Colorado, June 1955.

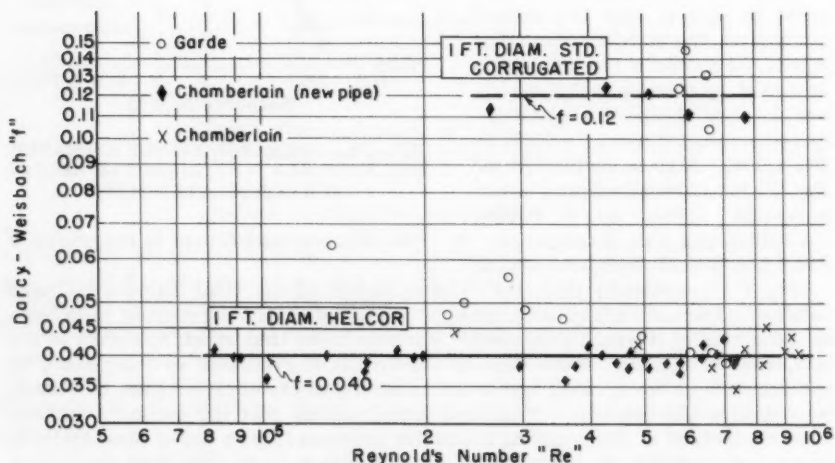
FIG. C3.—DARCY-WEISBACH COEFFICIENT VERSUS REYNOLD'S NUMBER FOR
1 FT. DIAMETER HELCOR AND STANDARD CORRUGATED PIPES.

TABLE 2.—SUMMARY OF TEST RESULTS FULL PIPE FLOW
1-FT-DIAMETER HELICAL CORRUGATED PIPE

V, in feet per second (1)	Q, in cubic feet per second (2)	S, in % (3)	Water Temperature in °C (4)	Re $\times 10^{-3}$ (5)	f (6)
1.59 ^a	1.25	0.25	16.5	135	0.0640
2.62 ^a	2.06	0.50	15.8	218	0.0478
2.76 ^a	2.17	0.59	16.7	235	0.0500
3.31 ^a	2.60	0.95	17.0	284	0.0559
3.58 ^a	2.81	0.96	16.2	302	0.0483
4.17 ^a	3.27	1.27	16.7	356	0.0470
5.73 ^a	4.50	2.20	17.0	493	0.0435
7.50 ^a	5.89	3.56	14.6	608	0.0408
7.97 ^a	6.25	4.02	15.9	666	0.0408
8.35 ^a	6.56	4.22	16.3	705	0.0390
0.98 ^b	0.75	0.062	15.2	82	0.0413
1.05 ^b	0.80	0.068	15.4	89	0.0396
1.10 ^b	0.84	0.074	15.0	91	0.0394
1.46 ^b	1.13	0.12	14.8	102	0.0368
1.63 ^b	1.27	0.17	14.6	132	0.0401
1.77 ^b	1.40	0.19	17.4	155	0.0390
1.88 ^b	1.47	0.21	14.4	153	0.0375
2.08 ^b	1.63	0.26	26.4	227	0.0446
2.16 ^b	1.70	0.29	17.4	190	0.0397
2.22	1.73	0.32	14.2	177	0.0412
2.45 ^b	1.92	0.37	14.0	197	0.0400
3.43 ^b	2.70	0.70	17.4	298	0.0384
4.10 ^b	3.25	0.94	17.3	358	0.0360
4.29 ^b	3.40	1.11	17.3	375	0.0385
4.45 ^c	3.53	1.18	27.0	485	0.0422
4.52 ^b	3.60	1.28	17.2	420	0.0404
4.80 ^b	3.80	1.50	17.1	395	0.0419
5.17 ^b	4.10	1.62	17.0	445	0.0391
5.41 ^b	4.30	1.74	17.0	470	0.0382
5.45 ^b	4.32	1.90	17.0	472	0.0412
5.80 ^b	4.50	2.00	17.5	510	0.0383
6.00 ^c	4.78	2.15	27.1	660	0.0382
6.23 ^b	4.95	2.39	14.0	500	0.0397
6.61 ^b	5.23	2.65	17.5	580	0.0391
6.80 ^b	5.43	2.80	13.8	548	0.0390
7.23 ^c	5.70	3.15	26.2	765	0.0418
7.24 ^b	5.82	3.23	17.5	635	0.0397
7.39 ^c	5.80	3.24	22.5	735	0.0350
7.40 ^b	5.90	3.20	13.6	580	0.0376
7.95 ^b	6.30	4.25	17.5	700	0.0433
8.10 ^b	6.42	4.29	13.4	640	0.0422
8.21 ^c	6.60	4.09	27.0	900	0.0411
8.40 ^c	6.70	4.43	28.5	950	0.0406
8.40 ^c	6.70	4.20	21.3	820	0.0392
8.46 ^b	6.80	4.20	17.4	740	0.0396
8.68 ^b	6.95	4.79	13.3	675	0.0409
9.20 ^b	7.35	5.20	13.1	720	0.0396
9.51 ^b	7.62	5.50	13.0	735	0.0392
9.72 ^c	7.63	5.60	17.0	840	0.0456
9.80 ^c	7.70	5.80	20.7	925	0.0438

^a Data from R.J. Garde, "Sediment Transport Through Pipes," Thesis in partial fulfillment of the requirements for the Master of Science Degree, Colorado State University, Fort Collins, Colorado, December 1956.

^b Data from A.R. Chamberlain, "Effect of Boundary Form on Fine Sand Transport in Twelve-Inch Pipes," Dissertation in partial fulfillment of the requirements for the Doctor of Philosophy Degree, Colorado State University, Fort Collins, Colorado, June 1955.

^c Data obtained by Chamberlain after pipeline had been used to transport 0.2 mm sand for several hundred hours.

^d Arithmetic average, all Chamberlain data, $f = 0.0400$.

Acknowledgements.—The basic research studies were supported by Armco Drainage and Metal Products, Inc., Tau Beta Pi Association (through a fellowship), and Colorado State University.

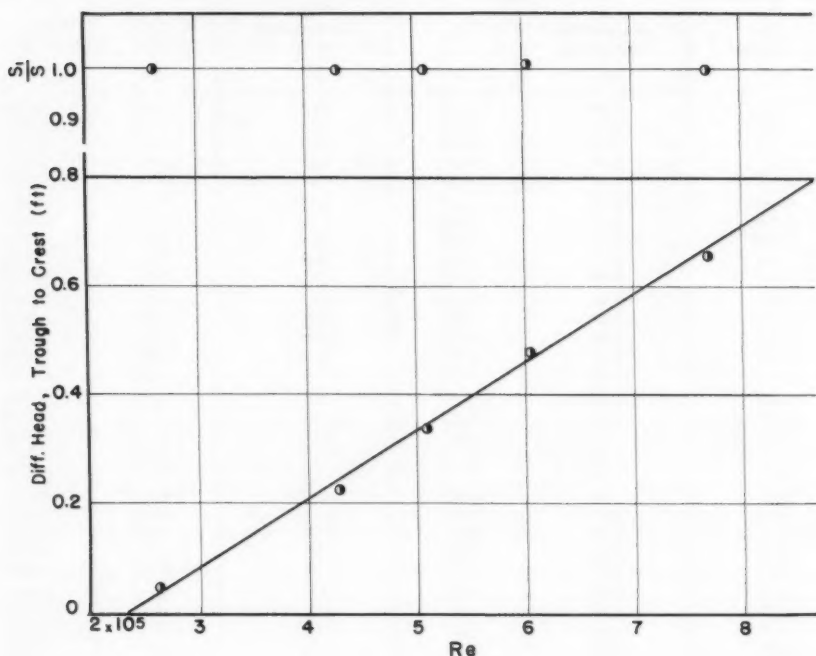


FIG. C4.—RATIO OF HYDRAULIC GRADIENT ON CORRUGATION HIGHS TO THAT OF CORRUGATION LOWS, AND DIFFERENCE IN HEAD BETWEEN HIGHS AND LOWS VERSUS REYNOLDS NUMBER FOR 12-IN. CORRUGATED PIPE.

NICHOLAS BILONOK.²—The most important contribution by the authors is the presentation of data from certain carefully conducted experiments on full size, large corrugated metal pipes, to determine friction factor " f " and absolute roughness " n ." The accuracy depends on a correct evaluation of " n ," the well known coefficient of absolute roughness in the Manning formula, which depends primarily on the surface condition of the perimeter of a pipe or open-channel.

Because of the complex nature of the friction factor " f " various attempts have been made to group all the roughness dimensions together into a single coefficient of absolute roughness " n " for simplicity.

The writer is particularly interested in this paper as he had independently investigated the relationship between friction factor " f " and absolute roughness " n ," for many years.

² Civ. Engr., Engrg. Div., Corps of Engrs., U.S. Army, Philadelphia, Pa.

The values of Q , S , H_f , L and D have been measured in experiments and the friction factor " f " has been found to be given by

$$f = \frac{H_f D^2 g}{L V^2} \dots\dots\dots (1)$$

or

$$f = \frac{8 g R S}{V^2} \dots\dots\dots (2)$$

The equation for the roughness coefficient " n " for the Manning formula is

$$n = \frac{1.486 R^{2/3} S^{1/2}}{V} \dots\dots\dots (3)$$

Eq. 3 may be written from the friction factor " f " by the relationship

$$n = \frac{R^{1/6} \sqrt{f}}{10.8} \dots\dots\dots (4)$$

Eq. 4 can be written as

$$f = \frac{116.57 n^2}{R^{1/3}} \dots\dots\dots (5)$$

or substituting $R = D/4$

$$f = \frac{185 n^2}{D^{1/3}} \dots\dots\dots (6)$$

From a review experiments data presented in the paper the writer has compiled Table 1 to relate the friction factor " f " to Manning's roughness coefficients " n " ranging from 0.010 to 0.025.

The values of " f " computed from Eq. 6 are in agreement with those obtained by experiments for pipes 3 ft, 5 ft, and 7 ft in diameter, and also with Table V in Reference 3.

Another quantity of importance that is the Reynold's number:

$$R = \frac{V D}{\nu} \dots\dots\dots (7)$$

The exact form and position of " f " versus Reynold's number for a given pipe diameter and roughness is not clear (Fig. 11), and close correlation requires more study.

The practical value of the Reynolds number is that it indicates the degree of turbulence in flowing water. For a given size of circular pipe the velocity is the major variable and the Reynolds number increases as the velocity of flow increases.

The author's Tables A and B indicate that the Reynolds number varies as velocity of flow only. The water temperature changes have no significance.

Investigations of friction losses for Part-Full flow, in Reference 3, have found that "For uniform subcritical flow, the Manning coefficient did not indi-

TABLE 1.—VALUES OF $f \times 10^3$

$\frac{n}{D, \text{ inches}}$	0.010	0.011	0.012	0.013	0.015	0.018	0.021	0.024	0.025
6	23.3	28.2	33.6	39.4	52.4	75.5	102.7	134.2	145.6
8	21.2	25.6	30.5	35.7	47.6	68.5	93.2	121.8	132.1
10	19.7	23.8	28.4	33.3	44.3	63.8	86.9	113.5	123.1
12	18.5	22.4	26.6	31.3	41.6	59.9	81.6	106.6	115.6
15	17.2	20.8	24.7	29.0	38.6	55.6	75.8	98.9	107.4
18	16.2	19.6	23.4	27.4	36.5	52.5	71.6	93.4	101.4
24	14.6	17.8	21.1	24.8	32.8	47.5	64.8	84.5	91.8
30	13.6	16.5	19.6	23.0	30.0	44.0	60.0	78.3	85.0
36	12.8	15.5	18.5	22.0	28.7	41.6	55.6	74.0	80.0
42	12.2	14.7	17.5	21.0	27.0	39.4	53.7	70.0	76.0
48	11.6	14.1	16.7	20.0	26.0	37.6	51.3	67.0	73.0
54	11.2	13.6	16.1	19.0	25.0	36.3	49.4	64.5	70.0
60	10.8	13.1	15.6	18.0	24.0	35.0	47.7	62.3	68.0
72	10.2	12.3	14.6	17.0	22.7	32.9	44.8	58.5	63.5
84	9.7	11.7	13.9	16.4	21.6	31.3	42.7	55.7	60.7
96	9.2	11.2	13.3	15.6	20.6	30.0	40.8	53.3	58.0
108	8.9	10.8	12.8	15.0	19.8	28.8	39.2	51.2	55.6
120	8.6	10.4	12.4	14.5	19.3	27.8	37.9	49.4	53.5
132	8.3	10.1	12.0	14.1	18.7	27.0	36.7	47.9	51.8
144	8.1	9.8	11.6	13.6	18.2	26.2	35.6	46.5	50.3

cate a systematic variation with the Reynolds number or pipe size within the accuracy of experimental data."

It can be recognized that several factors might have affected the accuracy of the test results and the computations.

REVISED COMPUTATION OF A VELOCITY HEAD WEIGHTED VALUE^a

Discussion by Steponas Kolupaila, Israel H. Steinberg
and William C. Peterson

STEPONAS KOLUPAILA.¹—Presentation of this paper proves that the problem of the correction factor, known as a Coriolis' coefficient, is still active, and still not entirely clear. The purpose of this factor is to correct the velocity head in the Bernoulli equation for different velocities across the section.

The basic expression given under 10, p. 72 is correct, except for an obvious typographic omission: ΔQ is to be understood instead of Q in the numerator. This formula can give an accurate solution when a stream is divided into a large number of small areas with measured velocities. This work is more reasonably performed by a graphical integration with a planimeter, eliminating those "astronomical" numbers. Application of this formula for several parts of a cross section with assumed coefficients is a half measure. The value obtained by the authors (1.19) seems to be too low for an open channel with overflowed banks.² The river gaging by two point or one point methods, although more than sufficient for the routine discharge measurement, are positively inadequate for the determination of the Coriolis coefficient. Two integrations are necessary in an open channel, along the verticals, and across the channel, and more points must be taken on every vertical.

ISRAEL H. STEINBERG,³ F. ASCE.—The sample computations presented in the article, and which are patterned after a procedure given in the Engineering Manual of the Corps of Engineers, still contain some of the objectionable, relatively large, figures required to obtain a relatively small value of velocity head. For this reason the writer has adopted a number of alternate procedures in order to further reduce the necessity of dealing with large and unwieldy values. One of the methods utilizes a weighted area curve for the section. The equation is:

$$A_w = A_c \left\{ \frac{1}{\left[\left(\frac{C_c}{C} \right)^3 + \left(\frac{C_o}{C} \right)^3 \left(\frac{A_c}{A_o} \right)^2 \right]^{1/2}} \right\} \dots \dots \dots (1)$$

^a September, 1959, by J.M. Lara and K.B. Schroeder.

¹ Prof. of Civ. Engrg., Univ. of Notre Dame, Notre Dame, Ind.

² "Open-Channel Hydraulics," by V.T. Chow. McGraw-Hill, 1959, p. 28.

³ Asst. Chf., Planning and Reports Branch, U.S. Army Engr. Dist., San Francisco, Corps of Engrs.

in which A_w is the weighted area, A_c and A_o are respectively, the channel and overbank areas, and C_c and C_o are, respectively, the channel and overbank conveyance factors of the form:

$$\frac{1.486}{n} \frac{A R^{2/3}}{L^{1/2}} = \frac{K}{L^{1/2}} \dots \dots \dots (2)$$

in which C is the total conveyance factor for the section ($= C_c + C_o$), and L is the reach length.

Eq. 1 was developed for flow line problems involving separation of the section into two parts; namely, channel and overbank. The equation can be readily expanded to include any desired number of components. Taking the reciprocal of Eq. 1 and squaring gives:

$$\begin{aligned} \left(\frac{1}{A_w} \right)^2 &= \left(\frac{1}{A_c} \right)^2 \left[\left(\frac{C_c}{C} \right)^3 + \left(\frac{C_o}{C} \right)^3 \left(\frac{A_c}{A_o} \right)^2 \right] \\ &= \left(\frac{C_c}{C} \right)^3 \left(\frac{1}{A_c} \right)^2 + \left(\frac{C_o}{C} \right)^3 \left(\frac{1}{A_o} \right)^2 \dots \dots \dots (3) \end{aligned}$$

Eq. 3 applies, also, for any number of components and can be written in the more generalized form of

$$\begin{aligned} \left(\frac{1}{A_w} \right)^2 &= \left[\left(\frac{C_1}{C} \right)^{3/2} \left(\frac{1}{A_1} \right) \right]^2 + \left[\left(\frac{C_2}{C} \right)^{3/2} \left(\frac{1}{A_2} \right) \right]^2 \\ &+ \left[\left(\frac{C_3}{C} \right)^{3/2} \left(\frac{1}{A_3} \right) \right]^2 + \dots \dots \dots (4) \end{aligned}$$

If L is constant throughout, the K -values can be substituted for corresponding C -values.

Eq. 4 can be further modified in more convenient forms for computing either (a) the weighted velocity head, h_v , or (b) the alpha coefficient, α , in terms of total section area. To obtain h_v , both sides of Eq. 4 are multiplied by $(Q/8.03)^2$, resulting in

$$\begin{aligned} h_v = \left(\frac{Q}{8.03 A_w} \right)^2 &= \left[\left(\frac{C_1}{C} \right)^{3/2} \frac{Q}{8.03 A_1} \right]^2 + \left[\left(\frac{C_2}{C} \right)^{3/2} \frac{Q}{8.03 A_2} \right]^2 \\ &+ \left[\left(\frac{C_3}{C} \right)^{3/2} \frac{Q}{8.03 A_3} \right]^2 + \dots \dots \dots (5) \end{aligned}$$

Application of Eq. 5 in computing the weighted velocity head is illustrated in Table 1 using the author's values for section 2. The values in Cols. 7 and 8 were obtained with simple settings of the slide rule.

TABLE 1

A_d	L	Q	K_d	$\left(\frac{K_d}{K}\right)^{3/2}$	$\frac{Q}{8.03 A_d}$	h_v
(3)	(4)	(5)	(6)	(7)	(8)	(9)
893	1110	10,800	66,400	0.255	1.511	0.149
589		$\frac{Q}{8.03} =$	67,000	0.258	2.295	0.352
150		134.8	5,240	0.006	9.000	0.003
594			26,300	0.062	2.280	0.020
2226			164,940			0.524

If the value of α is desired, Eq. 4 is multiplied by A^2 , the total sectional area, to give:

$$\alpha = \left(\frac{A}{A_w}\right)^2 = \left[\left(\frac{C_1}{C}\right)^{3/2} \frac{A}{A_1}\right]^2 + \left[\left(\frac{C_2}{C}\right)^{3/2} \frac{A}{A_2}\right]^2 + \left[\left(\frac{C_3}{C}\right)^{3/2} \frac{A}{A_3}\right]^2 + \dots \dots (6)$$

Computations are the same as for the example above except for Cols. 8 and 9. In Table 2 only Cols. 7, 8 and 9 are shown for brevity.

For the assumed value of $Q = 10,800$,

$$h_v = 1.424 \left(\frac{10800}{8.03 \times 2226}\right)^2 = 0.521$$

WILLIAM C. PETERSON,⁴ M. ASCE.—Computation of an approximation of the velocity head correction factor, α , by the fraction

$$\frac{K_1^3/A_1^2 + K_2^3/A_2^2 + \dots K_n^3/A_n^2}{(\sum K)^3/(\sum A)^2}$$

is essentially a simple slide-rule operation, quickly accomplished when one becomes familiar with the "feel" of the magnitude of the values involved. Techniques, which are largely personal, can abbreviate the consequent "astronomical values" and can also provide judgement in retaining the significant figures of the computation.

⁴ Hydr. Engr., U.S. Geol. Survey, Los Angeles, Calif.

TABLE 2

$\left(\frac{K_d}{K}\right)^{3/2}$	$\frac{A}{A_d}$	α
(7)	(8)	(9)
0.255	2.495	0.405
0.258	3.785	0.956
0.006	14.850	0.008
0.062	3.750	0.054
		1.423

In the Appendix of the author's paper the expression for friction head loss, h_f , is written as

$$h_f = f + h_{v1} - h_{v2} - h_t$$

where

$$h_t = m (h_{v1} - h_{v2})$$

The authors neglect to state, however, that the quantity in parentheses must always be positive. For an expanding reach, $h_{v1} > h_{v2}$, the quantity in parentheses is positive, and

$$h_t = m (h_{v1} - h_{v2})$$

Thus

$$h_f = f + h_{v1} - h_{v2} - m (h_{v1} - h_{v2})$$

or

$$h_f = f + (h_{v1} - h_{v2}) (1 - m)$$

If $m = 0.5$

$$h_f = f + (h_{v1} - h_{v2}) (0.5)$$

However, for a contracting reach, $h_{v1} < h_{v2}$, and the quantity in parentheses is negative. It is made positive in the following way:

$$h_t = - m (h_{v1} - h_{v2})$$

Then

$$h_f = f + h_{v1} - h_{v2} + m (h_{v1} - h_{v2})$$

or

$$h_f = f + (h_{v1} - h_{v2}) (1 + m)$$

If $m = 0.1$

$$h_f = f + (h_{v1} - h_{v2}) (1.1)$$

Expressing an approximation of α in terms of section properties, K and A , is essential to a direct method of determining the discharge of a stream by the slope-area technique. Consider the two-section reach 1-2 in Fig. D1.

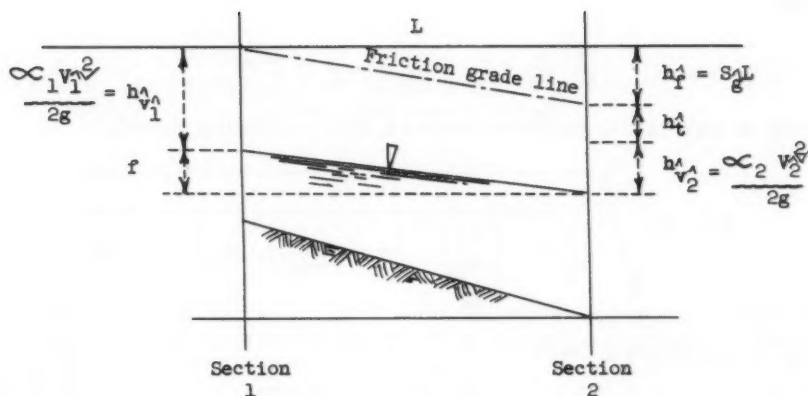


FIG. D1.

From continuity

$$Q_1 = K_1 S_1^{\frac{1}{2}} = K_2 S_2^{\frac{1}{2}} = Q_2$$

or

$$Q_2 = (K_1 K_2)^{\frac{1}{2}} (S_1^{\frac{1}{2}} S_2^{\frac{1}{2}})^{\frac{1}{2}}$$

that is

$$Q = K_g S_g^{\frac{1}{2}}$$

Also

$$h_f = f + h_{v1} - h_{v2} - h_t$$

For a contracting reach

$$h_t = -m (h_{v1} - h_{v2}) \quad \text{and} \quad h_f = f + (h_{v1} - h_{v2}) (1 + m)$$

For an expanding reach

$$h_t = m (h_{v1} - h_{v2}) \quad \text{and} \quad h_f = f + (h_{v1} - h_{v2}) (1 - m)$$

Moreover

$$h_f = S_g L = \frac{Q^2 L}{K_g^2} = \frac{Q^2 L}{K_1 K_2}$$

then

$$\frac{Q^2 L}{K_1 K_2} = f + (h_{v1} - h_{v2}) (1 \pm m)$$

which by algebraic manipulation becomes, for a contracting reach,

$$Q = K_2 \sqrt{\frac{f}{\frac{K_2}{K_1} L + \frac{\left(\frac{K_2}{A_2}\right)^2}{2g} \left[\alpha_2 - \left(\frac{A_2}{A_1}\right)^2 \alpha_1 \right] (1 + m)}}$$

and, for an expanding reach

$$Q = K_2 \sqrt{\frac{f}{\frac{K_2}{K_1} L + \frac{\left(\frac{K_2}{A_2}\right)^2}{2g} \left[\alpha_2 - \left(\frac{A_2}{A_1}\right)^2 \alpha_1 \right] (1 - m)}}$$

Modification of the area relationship by α values determines whether a reach is hydraulically contracting or expanding; that is, whether the weighted velocity head is increasing or decreasing in the downstream direction. If the quantity in brackets is positive, the reach is contracting; if negative, the reach is expanding.

Using data from the author's example, in the formula for a contracting reach, the discharge was computed as 10,700 cfs.

PERFORMANCE OF FLOOD PREVENTION WORKS DURING THE 1957 FLOODS^a

Discussion by Fred W. Blaisdell

FRED W. BLAISDELL,¹ F. ASCE.—Mr. Moore has presented interesting information on the operation of the SCS flood prevention structures in Texas, Oklahoma and Arkansas during the floods of April, May and June, 1957. According to Mr. Moore, the performance of the structures, designed for a 50-yr storm and subjected to greater than 100-yr storms, "... exceeded expectations. . ." The paper is devoted primarily to the performance of the emergency spillways. This is well because it is noted that "... the use of vegetated earth spillways for flood prevention structures . . . is a material divergence from the accepted principles employed by the engineering profession in past design."

Regarding the principal spillway, Mr. Moore notes that it is used to discharge water "... at a rate designed to prevent flooding of the stream channel below." A section through a principal spillway is shown in Fig. 3. It consists of a drop inlet, a barrel through the dam and a cantilevered outlet. The fact that the performance of the principal spillways is not mentioned is probably due to the fact that they performed as designed. The writer has been concerned with the hydraulics of the principal closed conduit spillways since 1941, and naturally was extremely interested in seeing how they came through these severe tests. In September, 1958, he visited a number of the structures. The visits confirmed the supposition that the structures did their intended job with complete satisfaction.

The practice of omitting stilling basins and simply cantilevering the pipe beyond the toe of the fill, as shown in Fig. 3, is a practice that is also "a material divergence from accepted principles." It was the performance of these cantilevered pipe outlets that the writer was particularly interested in evaluating. Compared to the size of the pipe, the storage between the crests of the principal spillways and the emergency spillways is large. At many of the structures, this storage pool was partially drawn down and refilled several times as a result of successive rains and many of the principal spillways ran at capacity for long periods of time. This gave maximum opportunity for the development of the scour holes anticipated to form at the pipe exit.

The performance of the cantilevered outlets was much better than the writer had expected. Typical scour holes are shown in Figs. B1 and B2. Fig. B1 is the outlet at Honey Creek Site No. 8E. There is a 17-in. concrete pipe through the fill and the cantilevered pipe is 18-in. corrugated metal. The

^a October, 1959, by Charlie M. Moore.

¹ Proj. Supervisor, Agricultural Research Service, U.S. Dept. of Agric., St. Anthony Falls Hydr. Lab., Minneapolis, Minn.



FIG. B1.

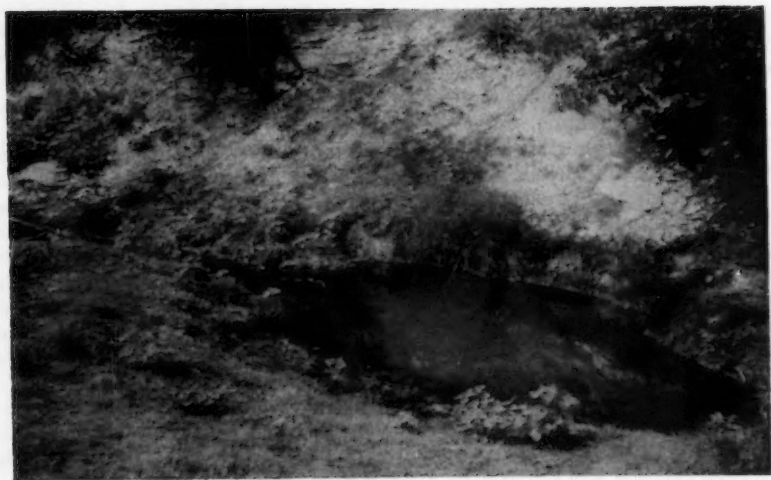


FIG. B2.

outlet ran for six weeks at full capacity. The 12-in. outlet pipe shown in Fig. B2 is for the Honey Creek Site No. 14. The depth of the pool in the scour hole is about 4 ft. It can be seen in Figs. B1 and B2 that the scour holes are small in size. At both sites, as well as all other sites seen, the scour hole terminated close to the end of the pipe. The bent shown in Fig. 3 is intended to support the pipe if the scour hole progresses upstream. The experience of the Texas structures indicates that this feature may not be necessary under the conditions existing there. However, it is not meant to imply that bents would be unnecessary in all cases.

The conclusion to be drawn from the Texas experiences is that a cantilevered outlet may be used in place of a stilling basin.

The writer has been looking at cantilevered outlets throughout the Midwest for a number of years with the expectation of finding some structures in which



FIG. B3.

the scour hole had reached a dangerous size. He recalls only perhaps two structures in which the scour holes looked as though they might eventually develop to a damaging size but which had not yet progressed to that point. One of these outlets is shown in Fig. B3. This is a 42-in. cantilevered outlet built in 1950. It carried a designed flow in 1952. This single flow formed the wide scour hole shown in the picture taken in 1954. Comparison with a picture taken in 1952, shows no visible increase in scour hole dimensions, possibly because additional capacity flows have not passed the structure. Both pictures show that the scour under the end of the pipe is insufficient to endanger it in any way. In fact, the scour has not undermined the pipe as far as the bent provided to support the pipe over the scour hole. In spite of the rather wide scour hole, the outlet performance must be classed as satisfactory. Whether

or not future capacity flows will increase the size of the scour hole to dangerous proportions is of course unknown at the present time.

The writer feels unsure of the performance of cantilevered outlets in spite of the fact that he has been unable to discover field evidence to back up his doubts. He feels there must be limitations to the size of cantilevered outlets that can be used with soils of different erodibilities in order to prevent the scour hole from endangering the dam. Here is a fertile field for research! Will the scour holes develop laterally, rather than in a dangerous upstream direction? Will upstream development of the scour hole take place after the lateral development has reached a certain extent? How long will it take to develop the scour hole? What is the effect of the bed material? What will be its ultimate size? Is this an economical self-formed stilling basin? These are questions that need answering.

Evidence collected by the writer indicates that this type of outlet structure has a definite usefulness. Lack of difficulty with it is such that there is no pressure from designers to conduct a thorough investigation into the characteristics and limits of applicability of the cantilevered outlet. However, the writer feels that research on the cantilevered outlet would be of major value to those who design and use this economical outlet structure.

THE VORTEX CHAMBER AS AN AUTOMATIC FLOW-CONTROL DEVICE^a

Discussion by Michael Amein

MICHAEL AMEIN,¹ M. ASCE.—The formation of vortices in flow, in hydraulic structures, has been found undesirable because of three principal features associated with flow containing a vortex. The first is the instability of the head-discharge relationship, resulting in an erratic hydraulic behavior. The second feature is the reduction of the discharge capacity, resulting in increased cost for the structure. The third feature is the possibility of damage from the erosive forces of the vortex motion. Such forces have been known to cause damage to inlet drop spillway structures at earth dams. The authors have described a case in which the presence of a vortex is not accompanied by any of the undesirable features mentioned. They have experimentally demonstrated that the erratic behavior can be eliminated by stabilizing the vortex in a vortex chamber. In the applications envisaged by them, the erosive forces are either insignificant or irrelevant, and the reduction in capacity would be used to advantage.

The type of vortex described by the authors usually results from flow through an orifice in the bottom of a tank, and is the closest approximation to a free vortex. In a free or mathematical vortex, the product of the tangential velocity and the distance from the core is constant. This conclusion can be derived by the application of Bernoulli's theorem along a streamline, and by equating the radial pressure differential to the centrifugal force. In this analysis the effects of the viscous forces are neglected. However, since in a real fluid vortex, viscous forces are present, the free vortex theory is not strictly applicable. Furthermore, from the principles of the free vortex motion as enunciated by Helmholtz, the free vortex consists of the same fluid particles. In the real fluid, the fluid particles travel in continuously shrinking orbits around the core and are discharged through the orifice. Thus, the particles constituting the vortex are constantly being renewed. It is the predominance of the tangential velocity in the vortex chamber that gives this vortex a free configuration and behavior, so that the application of the theory of free vortex in this case is probably correct for all practical purposes.

The authors have presented the results of their experiments in terms of a coefficient of discharge and a dimensionless parameter called the vortex number. This parameter contains the prime factors in the vortex motion. The writer would favor the publication of the observed data in conjunction with the results. A better understanding of the physical phenomena, the precision of

^a December, 1959, by R.C. Kolf and P.B. Zielinski.

¹ Hydr. Engr., Anderson-Nicholas & Co., Boston, Mass.

measurements and the quality of experimentation could be obtained from the observed data. Although the presentation of data in dimensionless parameters is a commendable procedure following the widespread acceptance of dimensional analysis, yet, this type of presentation, unless supplemented by observed data, freezes the data to a single interpretation. Furthermore, the data would not be accessible to support or refute other approaches that might come to light in the future.

The reduction in the discharge capacity of the orifice with the formation of the vortex can be said to be due to the conversion of part of the static head to the tangential kinetic energy in the vortex. In the vortex chamber, the introduction of flow at a tangential direction stimulates the formation of a vortex and tends to make the vortex a stable feature of the flow. The author's experiments indicate that the coefficient of discharge C is a unique function of the vortex number, from which the stability of the head discharge relationship is deduced.

In the following analysis, three new symbols are used in addition to the ones described by the authors: b = width of either of two inlet channels; W = velocity in the inlet channel at the entrance to the chamber; and θ = angle between the inlet channel alignment and the tangent to the chamber at the point of entry.

The velocity in the inlet channel is so small that the depression of the water surface due to the velocity head is neglected. Then inflow

$$Q = 2 H W b \quad \dots \quad (1)$$

The circulation is

$$\Gamma = \pi B W \cos \theta \quad \dots \quad (2)$$

But, from the definition of vortex number

$$\Gamma = D \sqrt{2 g H} \underline{V} \quad \dots \quad (3)$$

Therefore,

$$W = \frac{D \sqrt{g H} \underline{V}}{\pi B \cos \theta} \quad \dots \quad (4)$$

By substituting this value of W in Eq. 1,

$$Q = \frac{2 b D \sqrt{2 g H}^{3/2} \underline{V}}{\pi B \cos \theta} \quad \dots \quad (5)$$

The outflow is expressed as

$$Q = C A \sqrt{2 g H} \quad \dots \quad (6)$$

From Eqs. 5 and 6, it follows that

$$H = \frac{\pi B A \cos \theta}{2 b D} \frac{C}{\underline{V}} \quad \dots \quad (7)$$

Eq. 7 indicates that the head H in a given vortex chamber can determine the vortex number and the discharge. For the case in which $b = 6$ in., $B = 42$ in., $D = 3$ in. and $\theta = 20^\circ$.

$$H = 2.1 \frac{C}{V} \dots\dots\dots (8)$$

Between the values of $V = 0.8$ and $V = 2.0$, the relation between C and V , as determined from Fig. 2 is

$$V = 3.1 - 4.6 C \dots\dots\dots (9)$$

which, by substitution in Eq. 8, results in

$$C = \frac{2.1 H}{1.4 + 3.1 H} \dots\dots\dots (10)$$

If H is known, then C and Q can be readily calculated. Such a calculation is presented in Table 1.

It is interesting to compare the static head required to provide a given discharge when there is vorticity in the flow to the static head required to provide the same discharge when the vorticity is eliminated. For the latter case, a constant C equal to 0.686 is assumed. The static head required to provide a discharge Q can be determined from

$$H = \frac{Q^2}{2 C^2 A^2 g}$$

TABLE 1.—COMPUTATION OF DISCHARGE

H, in feet	C	Q, in cubic feet per second
0.8	0.43	0.15
1.0	0.47	0.18
1.2	0.49	0.21
1.4	0.51	0.24
1.6	0.53	0.26

The results of the computations are shown in Table 2.

It is seen from Table 2, that as the head on the orifice increases, a smaller portion of it is converted to vortex motion. Therefore, the vortex chamber

TABLE 2

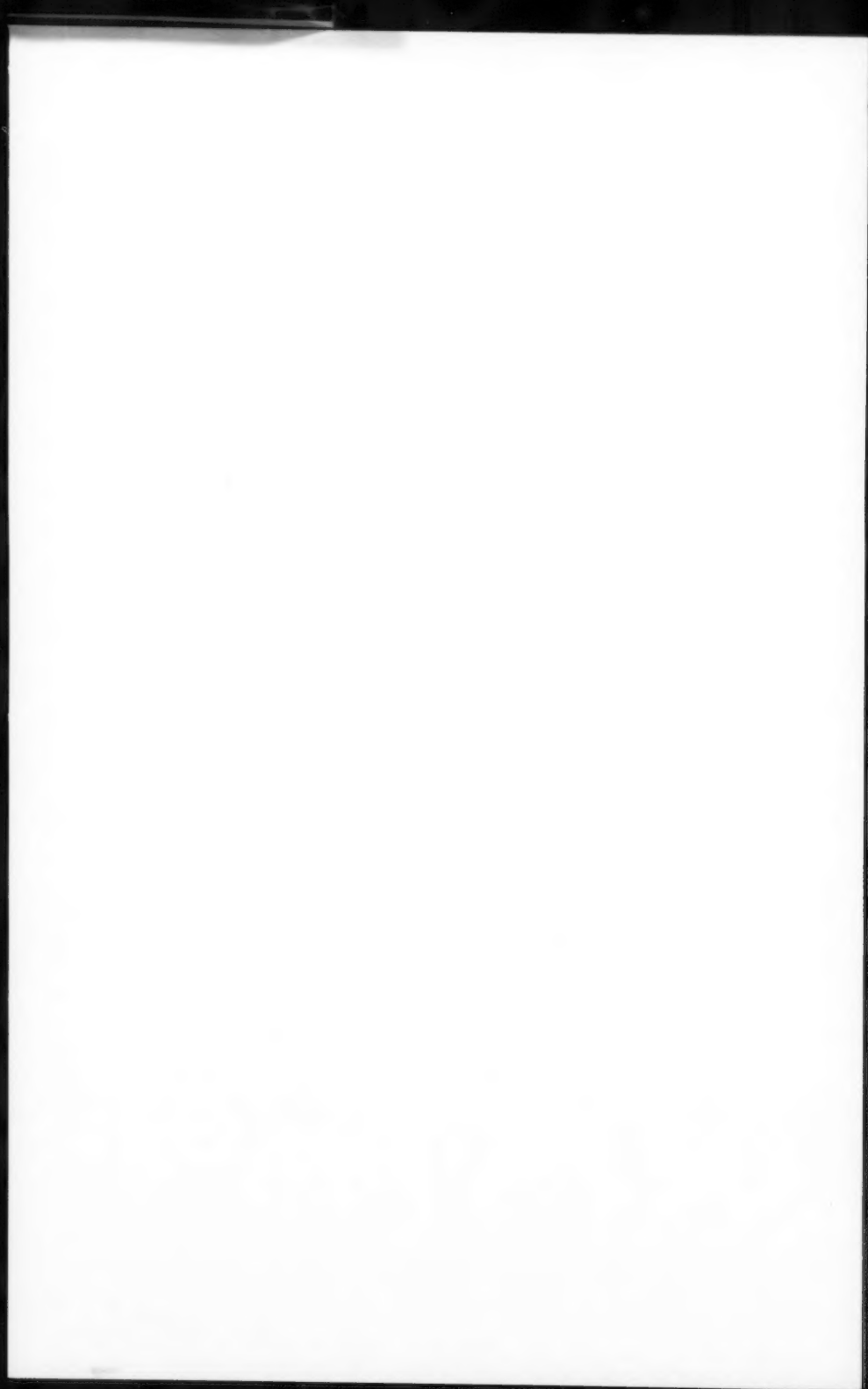
Q, in cubic feet per second	H with vortex, in feet	H with no vortex, in feet	E, in % ^a
0.15	0.8	0.4	47
0.18	1.0	0.6	39
0.21	1.2	0.8	32
0.24	1.4	1.1	24
0.26	1.6	1.2	22

^a E represents the percentage of the total static head lost in vorticity.

may not be useful as a flow control device, that would maintain a constant or a slowly rising discharge under a rising head. In fact, as has been experienced in drop-inlet spillway structures, a sufficiently high head will drown out the vortex motion and re-establish the normal orifice flow relationships. Many cases could exist in which the vortex chamber would perform no better than an orifice of smaller area. It appears that it is the range of heads and discharges within which flow takes place that determines whether the presence of a vortex in the flow can be permitted and whether the vortex can be made to perform a useful function. Further investigation might indicate that even in hydraulic structures it may be more feasible, in some cases, to stabilize the vortex than to make attempts towards its elimination. Research on vortex motion would not only open new fields for its application, but would also add to our basic understanding of the mechanics of flow.

References.—

- (1) "Fundamentals of Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, Dover Publications, Inc., New York, 1957.
- (2) "Hydraulics of Closed Conduit Spillways," by Fred W. Blaisdell, St. Anthony Falls Hydr. Lab., Minneapolis, Minn., 1958.



The first part of the paper discusses the
 importance of the study and the objectives of the research.
 The second part describes the methodology used in the study.
 The third part presents the results of the study.
 The fourth part discusses the implications of the findings.
 The fifth part concludes the paper.

The study was conducted in a laboratory setting.
 The participants were students from a university.
 The data was collected through a series of experiments.
 The results show that there is a significant difference
 between the two groups. The first group performed
 better than the second group. This suggests that the
 intervention had a positive effect on the outcome.

